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A STUDY OF THE LONGITUDINAL VIBRATION OF WIRES.

By George A. Lindsay.

I. Introduction.

THE elastic behavior of wires has been very carefully investigated by means of torsional vibrations. This method lends itself most readily, perhaps, to the study of internal friction, for the period may easily be regulated to any desired length, the amplitude may be made large, and, by suitable adjustment of the suspended mass, the free vibration may be continued for a comparatively long time.

Weber[1] was the first to observe that a vibrating body is damped in a vacuum, thus showing with certainty the existence of an internal cause of damping, and O. E. Meyer[2] made the first quantitative study of the phenomenon, by observing the damping of torsional vibrations.

Voight[3] measured the damping of flexural, as well as torsional vibrations, with the view of determining whether, according to Boltzmann's theory, the logarithmic decrement is independent of the period, as the latter shows it should be if damping is caused by the elastic after-effect; or whether there is really internal friction, in which case the logarithmic decrement should depend on the period. The results of his work were not entirely in favor of either theory. For flexular vibrations he found that, for copper and some other metals, the logarithmic decrement varied inversely as the period, thus supporting the internal friction theory, while with aluminum, cast iron, and cadmium the logarithmic decrement was much more nearly constant. In a few cases the decrement actually increased with increasing period, which is at variance with both theories. His conclusion was that in some cases the preponderant factor in damping is internal friction, in other cases it is the after-effect. Voight also found, as Schmidt[4] had done earlier, that the logarithmic decrement varies

[1] Pogg. Ann., 34, p. 247, 1835.
[2] Ibid., 113, pp. 77, 193, 1861.
[3] Abh. der Königl. Gesell. der Wiss. zu Göttingen, 38, 1892.
[4] Wied. Ann., 2, p. 48, 1877.

approximately as the square of the amplitude. Streintz,[1] however, found it to be independent of the amplitude in torsional vibrations, and also of the period, if the period were varied by changing the moment of inertia of the suspended mass. Guthe and Sieg[2] found certain platinum-iridium wires, when vibrated torsionally, to give a maximum value of the logarithmic decrement, and the position of the maximum on the decrement-amplitude curve depended on the initial amplitude. This work will be referred to more in detail later in comparing with the damping of longitudinal vibrations. Harris,[3] using bismuth wires, observed a decrease of the logarithmic decrement with decreasing amplitude, but no maximum or minimum values.

Other experiments, notably those of Streintz, Wiechert,[4] and Bouasse[5] and Carriere, have shown that the previous history of the wire very markedly influences the after-effect and the damping. While the after-effect has been carefully observed for longitudinal deformation, as far as the writer has been able to find, little or nothing has been done on the problem of the quantitative measurement of the damping of longitudinal vibrations.

The present investigation is an attempt to measure as accurately as possible the period of longitudinal vibrations of wires, to consider possible causes of difference between this and the theoretical period, and to measure the damping of the vibrations. Both static and dynamic observations will be given which indicate that the modulus of the wires is not exactly a constant with varying extension. An expression for the period involving the amplitude will be derived, which shows that this variation of the modulus cannot account for the observed variation of the period with amplitude. It will also be shown that, with the ordinary assumptions, the variation of the logarithmic decrement with amplitude is not sufficient to explain the variation of the period.

II. Elongation of the Wires.

Apparatus.

Four wires were used; one each of copper, steel, phosphor-bronze, and platinum-iridium. The steel wire was new piano-wire; the copper, a piece from a spool of commercial wire. Three of the four wires were about 230 cm. long, while the fourth, the platinum-iridium, was only about

[1] Wien. Ber., 69, II. Abt., p. 337, 1874: 80, II. Abt., p. 397, 1879.

[2] Phys. Rev., Vol. 30, No. 4, 1910.

[3] Phys. Rev., Vol. 35, No. 2, 1912.

[4] Wied. Ann., 50, p. 546, 1893.

[5] Ann. de Fac. des Sci. de Toulouse, II., 431; III., 217; IV., 357. Ann. de Chimie et de Physique, 8me Serie, t. 14, p. 190.

140 cm. long. No attempt was made to take account of previous treatment of the wires, since the adjustment for vibration was rather difficult, and it was usually necessary to vibrate the wire for some time before measurements could be made. Excepting the platinum-iridium, which contained 40 per cent. of iridium, and was prepared by Dr. Heraeus, of Hanau, the composition of the wires was not well known, and it is not to be expected that the results are rigidly characteristic of these metals. In fact, it is known that hardness has a great influence on the internal friction of wires. Annealed wires exhibit greater damping than tempered ones of the same material.

A heavy iron bracket was fixed rigidly to the brick wall of the laboratory. A strap of iron was bolted firmly to the front of the bracket; but this was not used as a clamp, for clamping the wire would flatten it, and weaken it at the point where it entered the clamp, besides rendering uncertain the location of the upper end of the vibrating length. Instead, a small slot, only large enough to admit the wire easily, was sawed in the iron strap. This slot was filled with solder, and the upper end of the wire was soldered in, thus attaching the wire very firmly to the support. In order to keep the temperature uniform and fairly constant, the wire was enclosed in a wooden box, the internal cross-section of which was about 16 cm.^{2} The upper end of the box was lightly covered with cotton batting to prevent air currents passing through, yet not so as to interfere with the vibrations. Two thermometers were inserted through the side of the box, one near the top and the other near the bottom.

A fan was installed near the box to keep the air in the room in circulation. Without the fan the temperature of the upper part of the box sometimes became as much as 3 degrees higher than the lower end, especially in colder weather, when the room was heated by pipes near the ceiling. When the fan was working the upper and lower parts of the box differed usually by not more than 0°.1 C. A variation of one or two degrees has very little effect on the period of vibration, since the modulus changes only slightly with the temperature; but in determining statically the elongation due to a certain load, it was very important that the temperature should not change between readings.

The following precautions were taken to straighten the wires before using. The steel and phosphor-bronze wires were suspended and stretched for about one day by ½ the maximum load which they were required to carry. The copper wire was very soft, and was straightened by passing it through a drawplate, using a hole the size of the wire. This straightened the wire without drawing it any smaller. The platinum-iridium wire was moderately loaded and annealed by passing through it an electric current sufficient to raise it to a yellow heat.

Instantaneous Recovery.

In order to make a comparison of static and dynamic moduli, the simple method of measuring the elongation directly was chosen. Consider a wire loaded as in Fig. 1. Let P be the position of the lowest point of the wire when at rest. From the elastic forces called out by the distortion, P has a tendency to return to some point O. This point may not be at all the position of P before the load was imposed. It does not even remain constant when P moves up and down. The variation of O is due to two things: (1) the elastic after-effect; (2) the heating effect of varying the length of the wire. When the wire is at rest, (2) is not to be considered; therefore, disregarding this for the present, let us call $OP = e$ the instantaneous recovery. This is evidently the distance on which the restoring force depends, and this is the distance it is desired to measure.

Fig. 1.

Lever Systems for Measuring Recovery.

In order to determine changes occurring very soon after unloading, the device shown in Figs. 2, 3, and 4 was used. Figs. 2 and 3 show two sections at right angles to each other. Fig. 4 shows one of two similar

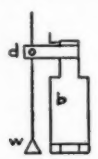

Fig. 2.

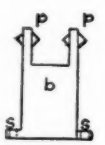

Fig. 3.

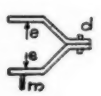

Fig. 4.

levers, made each of two stips of brass bent and soldered together along one half their length. Small conical hollows were made at *ee*, Fig. 4, to receive the points of *p* in Fig. 3. The arms of the lever were bent just far enough apart so they would hold on the axle *p* without play, and with little friction. The lever was thus free to move about the axis of *p* in a vertical circle. *L*, Fig. 2, is this lever seen from the side. *d* is a small closely fitting pin to one end of which the wire *W* was lightly soldered. *m* is a small mirror with its plane at right angles to the lever, and containing the axis of *p*. One of the levers was used for the wire under test, the other for a comparison wire hung about 2 cm. from the first. The mirrors on the two levers were placed so close together that by means of a single telescope two scales could be viewed at the same time, one reflected from each mirror.

Wire No. 2 indicated any sagging of the bracket, and change of temperature. It was found that with the fan running and the box in position the temperature change during the readings necessary for a single determination of the elongation was ordinarily too small to be observed. For measuring the elongation the mass of figure 1 was replaced by a scale pan.

A mass was placed on the pan and after some time a reading was taken on both scales, and the mass was immediately removed. Then readings were taken on the wire under test at the end of 5 sec., 15 sec., 1 min., 2 min., and 3 min. Two or three readings on the comparison wire gave corrections to be applied for sagging of the support, and for temperature changes. The comparative rigidity of the lever system allowed a reliable reading to be taken 5 sec. after removal of the mass. The results are shown for the copper wire in figure 5. The numbers in parentheses refer to different masses removed from the pan as shown in the accompanying table.

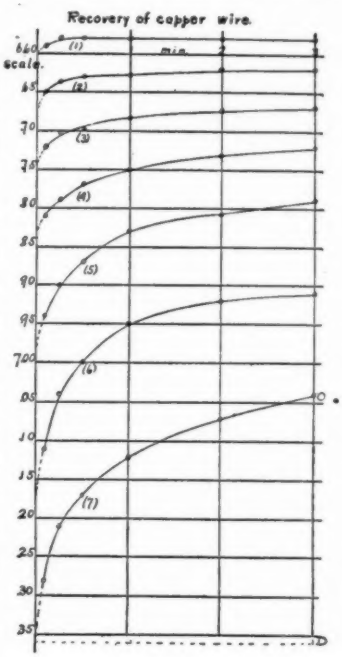

Fig. 5.

Copper.

	Mass, gms.
(1)	461
(2)	930
(3)	1,401
(4)	1,872
(5)	2,343
(6)	2,815
(7)	3,295

Each mass was left on the pan 2 min. The relative positions of the curves in Fig. 5 show how slowly the copper wire recovered from strain. The after-effect and the heating effect are both present in these curves. The curve for a time interval less than 5 sec. is unknown, but producing it as the observed part seems to indicate we would have an intersection on the scale axis as shown by the dotted line.

Owing to the various quantities upon which the computed elongation depends when deduced from the scale readings, and the possibility of error, especially in the motion of the pin *d*, shown in Fig. 2, it was thought

best to depend on the above apparatus only for the small changes taking place in the first minute or two, and to measure the entire recovery of the wire simply by means of a micrometer microscope focused directly on a point of the wire. Then the correction shown by Fig. 5, and that for the sagging of the bracket were applied afterward.

For all except copper it was seen that in one half min. to one min. the wire had entirely recovered from the strain. The microscopic readings for these could therefore be taken soon after removal of the load. With the copper they were all taken long enough after release so that the change in length had nearly ceased, but the time of each reading was noted, and the proper correction applied from the plots.

Instantaneous Recovery Independent of Period of Deformation.

It was found by means of the lever system that the length of time the load remained on the pan did not influence the instantaneous recovery. The following table shows the instantaneous recovery for various periods of deformation of the copper wire. t is the length of time the weight remained on the pan, e the instantaneous recovery in cm. of the scale. It is the difference between the intersection of the curve with the scale axis (see Fig. 5) and the reading just before release.

TABLE I.

Load = 2 Kg.

t min.	e.
1	11.25
5	11.23
15	11.20
30	11.22
60	11.22
30	11.23
15	11.21
5	11.20
1	11.23

This indicates that although the wire may suffer a gradual change when distorted by a constant force, a given point of the wire has always the same instantaneous recovery.

Since the change observed by aid of the levers and mirrors after the removal of the load is small compared with the total recovery, we may compute the change s in the length of the wire, corresponding to the distance OD shown on the plot of Fig. 5, by the equation

$$s = \frac{1}{2}\frac{r}{a} OD, \qquad (1)$$

where r is the radius of the lever arm and a is the distance from mirror to scale. r was determined by measuring, with a microscope, the diameter

of the circle in which the pin d moves when the lever is rotated about the axis. r was thus found to be 2.124 cm.

Temperature Change Due to Loading and Unloading.

The formula for temperature change given by Sir William Thomson[1] is

$$t = \frac{ATa}{\omega c}\Delta p, \tag{2}$$

where A = the reciprocal of Joule's equivalent.
T = abs. temp.
α = coeff. of linear expansion.
ω = linear density of the wire.
c = specific heat.

Edlund[2] found, experimentally, that this formula gave too high a value for t, and according to his results $A = \frac{1}{68270}$ when Δp is in gm.

When the load is suddenly removed the wire is heated, and as it cools it contracts, which contraction added to the after-effect gives the curves in Fig. 5. To compute the part due to the cooling of the wire we have the change of length due to a change in temperature t,

$$\Delta l = \frac{AT\alpha^2 L}{\omega c}\Delta p. \tag{3}$$

If we take L = 200 cm.,

and Δp = 3,000 gm., Table II. shows the Δl of equation (3) for each of the four wires. Edlund's value of A is used.

TABLE II.

Wire	Δl cm.
Steel	0.0010
Copper	0.0004
Phos. br.	0.0020
Pt.-ir.	0.0008

The instantaneous recovery of the wires is given by

$$e = e' - s + \Delta l, \tag{4}$$

where e' is the elongation as measured by the microscope directly. Δl must be determined for the particular length used in obtaining s; then if this is not the same as the length used in measuring e', $s - \Delta l$ must be reduced to that length before subtracting from e'. For the phosphor-bronze and platinum-iridium $s = \Delta l$ as nearly as the plots will show. For steel, s was a little more than twice Δl, and for copper, nearly ten

[1] Math. and Phys. Papers, Vol. 3, p. 66.
[2] Pogg. Ann., 126, p. 539, 1865.

times Δl. This indicates that phosphor-bronze and platinum-iridium have no after-effect, yet this supposition does not agree with the results of timing. Reasons will be given later for supposing that all the wires have an elastic after-effect.

Variation of Instantaneous Recovery with Temperature

To find how the instantaneous recovery varies with the temperature, which is equivalent to finding the variation of the modulus with temperature, the room was kept successively at ten different temperatures ranging from 15°.7 to 26°.4 C., and the instantaneous recovery was determined for each temperature for all the masses concerned in Fig. 5. This test was made for copper only, for the variation of the modulus with temperature is much smaller for the others. According to Pisati it is only one part in 10,000 per degree for steel at temperatures from 0° to 50°. With copper, however, it is large enough to be detected over the range of only 10° employed here, and by plotting instantaneous recoveries and temperatures the equation

$$e_t = e_{23}[1 + 0.0009\ (t - 23)] \qquad (5)$$

was found for the relation between the recovery and the temperature.

Since the word elongation is not to be used in any other sense in this paper, "elongation" will henceforth be used to denote the instantaneous recovery.

Rapidity with Which the Wire Regains Normal Temperature.

The heat evolved by the removal of the load is soon dissipated. A test was made on the copper wire, which, with a diameter of 0.5 mm., was much the largest wire of the four, to see how much time was required for the wire to regain its original temperature. An iron-constantan thermocouple was connected to the wire by separating one of the junctions, passing the separated elements through small holes in the side of the box, and soldering them as lightly as possible to the copper wire, a few cm. apart. The junctions were covered with a little cotton batting so they might cool at most no faster than the remainder of the wire. The other junction of the thermocouple was kept at a constant temperature outside the box. The cross-section of the thermocouple wire was about 1/30 that of the copper wire to which it was attached.

When a mass of 2 kg. was removed from the pan, the deflection of a galvanometer in the thermocouple circuit had practically all disappeared in one minute, showing that in that time the wire had regained its normal temperature. Since the other wires were of less than one half the diameter of the copper wire, the rise in temperature would no doubt disappear in a much shorter time for them.

Results for Elongation.

In Table III. are given the elongations for the various wires. The values of e for copper were all reduced to 23° by formula (5). While the entire set of elongations for copper was taken at 21°, the wires were vibrated at higher temperatures, and it was for comparison with the vibration results that the reduction was made. For all the other wires the variation of modulus with temperature was neglected.

M and e are corresponding masses and elongations, the zero for each being taken with the pan alone suspended.

M includes one half the mass of the wire.

Mass of the pan = 223 gm.

L' = length for which the elongation was measured.

L = vibration length.

e = elongation for the length L.

D = diameter of the wire.

TABLE III.

Copper.

L' = 209.7 cm.
L = 232.1 cm.
D = 0.586 mm.
Temp. 21°.0

M Gm.	e Cm.	e_{23} Cm.	M/e.
957	0.0821	0.0822	11,640
1,387	0.1182	0.1184	11,710
1,818	0.1560	0.1563	11,630
2,248	0.1941	0.1945	11,560
2,679	0.2315	0.2320	11,550
3,101	0.2690	0.2695	11,510

Steel.

L' = 154.1 cm.
L = 231.1 cm.
D = 0.217 mm.
Temp. 24°.2

M Gm.	e Cm.	M/e.
1,000	0.2874	3,478
1,500	0.4309	3,480
2,000	0.5766	3,467
2,500	0.7213	3,465
3,000	0.8675	3,457

Phosphor-bronze.

L' = 187.6 cm. D = 0.239 mm.
L = 231.5 cm. Temp. = 22°.5.

M Gm.	e Cm.	M/e.
953	0.4138	2,303
1,383	0.6023	2,296
1,814	0.7912	2,293
2,244	0.9816	2,286
2,675	1.1742	2,278

Platinum-iridium.

L' = 140.5 cm. D = 0.206 mm.
L = 144.6 cm. Temp. = 22°.5.

M Gm.	e Cm.	M/e.
953	0.1461	6,523
1,383	0.2119	6,527
1,814	0.2780	6,525
2,244	0.3448	6,508
2,675	0.4137	6,466
3,097	0.4812	6,436

M/e has no significance for the material of the wire, since it contains the length and diameter, but as long as these are constant it serves to indicate whether Young's modulus is constant or not. Owing to the shortness of the wires used, no great degree of absolute accuracy is claimed for the above elongations, and also, since the computation of the period of vibration does not demand it Young's modulus has not been

computed. Its absolute value is of little importance here, but its variation is of great importance.

III. Determination of Frequency of the Vibrations.

Apparatus.

The pan was removed from the wire, and in its place was soldered a brass disk 6½ cm. in diameter, provided with threads on the circumference, so that over it might be screwed a hollow cylindrical shell about 14 cm. long. Inside this shell could be attached five other solid disks, making six different masses which could be used in vibrations. The shell was closed at the bottom, so that for all masses used, the same surface was exposed to the air and the damping due to the air friction was, therefore, constant.

From the lower end of the cylinder a brass rod 8 mm. in diameter projected vertically downward an additional 10 cm., and carried on its lower end a small soft iron armature. The sections of the mass separated from

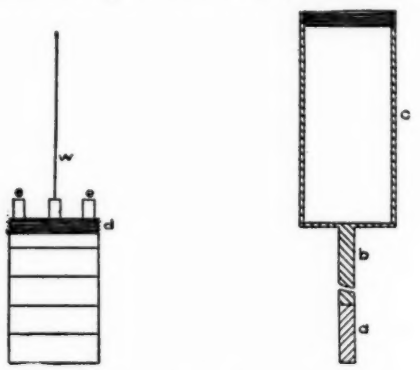

Fig. 6.

a, soft iron armature; *c*, cylindrical shell; *d*, upper disk which screws into the shell.

the cylinder are shown in Fig. 6. The electric circuit was arranged, as in Fig. 7, so that the wire was self-driven after the manner of the electrically driven tuning fork. The switch L closed both circuits at once. The vibration could be started by raising or lowering the adjustable cup g until, when the proper height was reached, the vibration would start and the amplitude could be controlled by the rheostat.

A tuning fork supplied with adjustable masses for regulating the period, and carrying a lens on one prong, was arranged to vibrate horizontally in front of the wire, so that when a drop of mercury was placed on the wire a Lissajous' figure could be viewed through the vibrating lens by means of a microscope. A phonic wheel was included in the circuit with the tuning fork, so that the fork could be accurately rated while

the wire was vibrating. The vibration was so slow that the time required for an elastic pulse to travel the length of the wire was very short compared with the period of the wire, therefore we may consider the wire to be uniformly strained throughout its entire length at all times.

Measuring the Free Period.

This, of course, was a forced vibration of the wire, and the period was therefore slightly less than that of the free vibration. It was found, however, that by adjusting so that the fork was slightly too slow, then

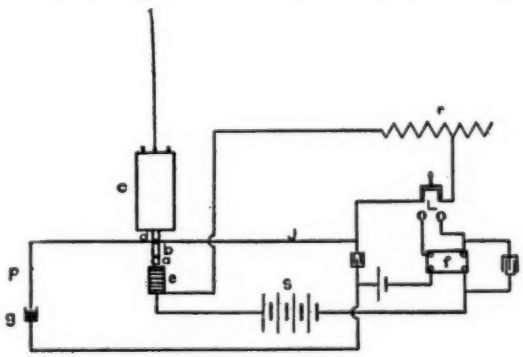

Fig. 7.

e, electromagnet actuated by battery *s*; *f*, relay; *g*, mercury cup; *j*, fine wire soldered to moving system at *d*; *p*, stiff platinum-tipped wire for contact with mercury at *g*.

shutting off the current driving the wire, the free period could be obtained with considerable accuracy.

The free vibration could be observed easily for ¾ min. with the copper wire, about 4 min. with the platinum-iridium wire, and 5 min. with the steel and phosphor-bronze. In case the period varied with the amplitude, as it did with the copper and platinum-iridium, and also to a slight extent with the phosphor-bronze, the fork was set too slow for small amplitudes, and too fast for large ones. By means of the micrometer microscope the amplitude for which the Lissajous' figure was at rest could be satisfactorily determined. This, however, necessitated changing the rate of the fork, and so rating again by the phonic wheel for every amplitude measured, an expenditure of time which was partly avoided by counting the number of seconds elapsing during one cycle of the Lissajous' figure and taking for the mean amplitude, the average of the amplitudes at the beginning and the end of the count.

Relation of Frequency to Amplitude.

For the steel wire no variation of the frequency with amplitude could be detected. The phosphor-bronze showed a very slight increase in

frequency with decreasing amplitude, the variation for platinum-iridium was more marked, and the copper wire showed the greatest variation of all. In all cases the plot of frequency and amplitude was practically a straight line, which, produced to the frequency axis, gave the frequency

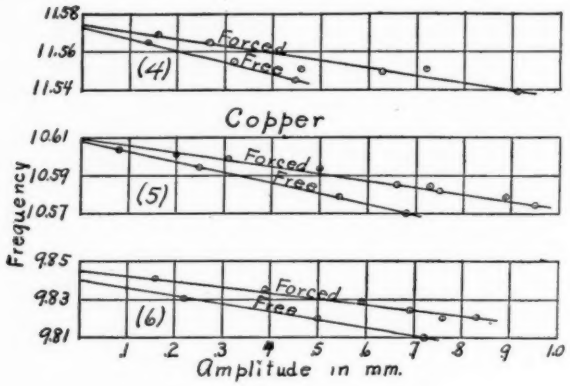

Fig. 8.

for infinitely small vibrations. The equation connecting frequency and amplitude is $n = n_0 - bA$, where b is a positive number. The relation is shown graphically for part of the observations in Figs. 8 and 9. The numbers (4), (5), (6), in parentheses, refer respectively to the largest

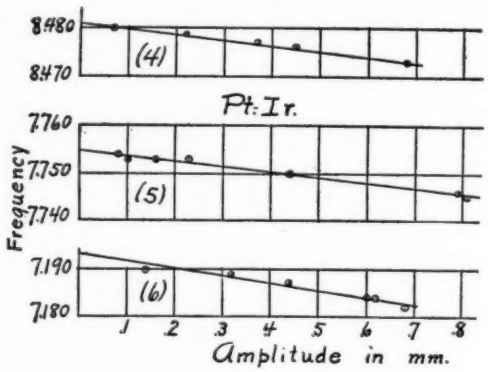

Fig. 9.

three masses given in Tables V. and VI., in order of increasing magnitude. The cause of this variation of period with amplitude will be discussed later.

Free and Forced Vibrations.

In Fig. 8 for copper are included the plots of the forced frequency. It is seen that this also gives a straight line of nearly the same slope as that of the free vibrations, and the forced period differs very little from

the free. That there is not a greater difference is due to the fact that the relay included in the circuit aided the induction in delaying the pulse given by the electromagnet. If the pulse were given just at the instant when the armature reached the middle of its downward motion, there would be no difference of the free and forced periods.

Although the force acting on the armature is not simply harmonic with respect to the time yet it may be represented by a Fourier's series, and it may be shown from the difference between the forced and the free periods that for small amplitudes the fundamental of the Fourier's series is in the neighborhood of 65° behind the motion of the armature.

Table IV. is given as an illustration of the manner of timing.

TABLE IV.

Copper.

M = 2,678 gm. ρ = 5/1. Temp. = 21°.9 C.

N = frequency of fork.
n = frequency of wire.
$\rho = N/n$.

N	Forced.		Free.		Sec. per Cycle.	Wire Slow or Fast.
	Amp. (Mm.)	n	Amp. (Mm.)	n		
			0.08	10.604		
53.020	0.20	10.602			102	slow
	0.31	10.598			34	slow
			0.25	10.593		
	0.50	10.593			∞	——
52.964	0.66	10.586			31	slow
	0.73	10.583			21	slow
			0.54	10.578		
	0.75	10.582			44	fast
52.888	0.89	10.578			∞	——
	0.95	10.574			50	slow
52.860			0.68	10.572		

The 6th column shows the number of seconds required for the Lissajous' figure to pass through one cycle, while the last indicates whether the wire was faster or slower than 1/5 the rate of the fork.

Tables V. and VI. show in condensed form the free amplitudes and frequencies for the various masses.

Such tables are not given for phosphor-bronze and steel for reasons already stated.

TABLE V.

Copper.

Mass, Gm.	Amp., Mm.	n	ρ	Temp., C.
	0.10	17.700		
956	.15	17.693	5/2	23°.7
	.33	17.676		
	.17	14.715		
1,386	.32	14.696	3/1	23°.8
	.59	14.672		
	.19	12.853		
1,817	.24	12.848	7/2	24°.0
	.40	12.839		
	.14	11.565		
2,247	.32	11.555	4/1	24°.0
	.45	11.545		
	.08	10.604		
	.25	10.593		
2,678	.54	10.578	5/1	21°.7
	.68	10.572		
	.22	9.831		
3,100	.50	9.818	5/1	23°.8
	.72	9.810		

Variation of Frequency with Mass.

Since, if Hooke's Law holds the period is proportional to the square root of the mass, we should have the product of mass and the square of the frequency equal to a constant. From Table III., the ratio M/e is not a constant. It decreases with increasing mass, except for the first two masses of the three wires, copper, steel and platinum-iridium. It would not be concluded from this alone that the elongation is not proportional to the weight removed, for there is considerable uncertainty about the point to which the curves of Fig. 5 should be produced before they reach the axis of ordinates. However the results of timing also show that M/e is not a constant, for Mn^2 is not constant, except for the case of copper. Table VII. shows the masses, frequencies, and Mn^2, for each of the four wires.

The n for the copper and the platinum-iridium is obtained from Figs. 8 and 9. It is the n indicated by the straight line, for the amplitude zero. For copper a small correction to n was made to reduce it to 23° temperature, the same temperature as that for the elongations of Table

TABLE VI.

Platinum-iridium.

Mass, Gm.	Amp., Mm.	n	ρ	Temp., C.
	0.08	13.056		
	.20	13.053		
953	.44	13.046	3/1	23°.0
	.58	13.044		
	.80	13.037		
	0.09	10.836		
1,383	.47	10.828	4/1	23°.2
	.75	10.824		
	0.05	9.450		
	.12	9.449		
1,814	.21	9.447	5/1	24°.4
	.66	9.442		
	.98	9.439		
	0.07	8.480		
	.22	8.479		
2,244	.37	8.477	5/1	24°.5
	.45	8.476		
	.68	8.473		
	0.08	7.754		
	.10	7.753		
	.16	7.753		
2,675	.23	7.753	5/1	24°.6
	.44	7.750		
	.79	7.746		
	.81	7.745		
	0.14	7.190		
	.32	7.189		
	.44	7.187		
3,097	.60	7.184	6/1	24°.6
	.62	7.183		
	.68	7.182		
	.78	7.181		

III. Since $1/n \propto \sqrt{e}$, one half the correction in per cent. which was made for e, was made here for n: or we might use a formula similar to equation (5)

$$n_t = n_{23}[1 - 0.00045(t - 23)]. \qquad (6)$$

For the other three wires the correction was neglected for reasons already stated.

TABLE VII.

Copper.						Steel.			
M[1] Gm.	n	Temp.	Corr.	n_{22}.	Mn^2.	M Gm.	n	Temp.	Mn^2.
956	17.710	23°.7	+0.006	17.716	3.001×10^5	954	9.525	21°.5	8.655×10^4
1,386	14.732	23°.8	+0.006	14.738	3.011	1,384	7.908	21°.6	8.655
1,817	12.865	24°.0	+0.006	12.871	3.010	1,815	5.903	21°.7	8.649
2,247	11.573	24°.0	+0.005	11.578	3.012	2,245	6.199	23°.1	8.627
2,678	10.607	21°.7	−0.005	10.602	3.010	2,676	5.675	23°.0	8.618
3,100	9.841	23°.8	+0.004	9.845	3.005×10^5	3,098	5.268	22°.7	8.598×10^4

Phosphor-bronze.				Platinum-iridium.			
M Gm.	n	Temp.	Mn^2.	M Gm.	n	Temp.	Mn^2.
953	7.767	21.3	5.749×10^4	953	13.058	23.0	1.625×10^5
1,383	6.437	21.8	5.731	1,383	10.837	23.2	1.624
1,814	5.611	21.9	5.711	1,814	9.450	24.4	1.620
2,244	5.038	22.0	5.696	2,244	8.481	24.5	1.614
2,675	4.594	21.3	5.646×10^4	2,675	7.755	24.6	1.609
				3,097	7.193	24.6	1.602×10^5

In Table VII. all the wires but the copper show a steady decrease in Mn^2 with increase of M. The decrease is very small, or zero, between the first two masses, and it has been noted that there was, in general, no decrease of M/e in Table III., until the second mass was reached.

Relation between M and e.

For all except the small masses the results of both the direct measurement of elongation and of the timing, indicate for steel, phosphor-bronze, and platinum-iridium a greater elongation than that demanded by Hooke's Law. We will therefore assume the relation

$$M = \alpha e + \beta e^2, \tag{7}$$

and deduce the period which would result from oscillations along the curve represented by this equation. Since M/e decreases with increasing M, β will evidently be negative. It is more logical, perhaps, to write

$$e = aM + bM^2,^2 \tag{8}$$

but for solution of the differential equation given later it is simpler to have M expressed in terms of e, as in (7).

[1] M includes one third the mass of the wire, which addition was negligible for all except copper.

[2] J. O. Thompson found the formula $e = aM + bM^2 + cM^3$ to satisfy his observations in static experiments with wires. See Amer. Jour. Science, Vol. 43, p. 32, 1892.

IV. Derivation of the Period.

If we assume small vibrations of the wire, the period, neglecting the damping term, is known to be

$$T = 2\pi \sqrt{\frac{M}{\frac{dM}{de} g}}. \tag{9}$$

From (7) $dM/de = \alpha + 2e\beta$, whence

$$n = \frac{1}{2\pi} \sqrt{\frac{g(\alpha + 2\beta e)}{M}}. \tag{10}$$

Since the period has been shown by Figs. 8 and 9 to depend on the amplitude, the following derivation is given to show whether vibration along the curve represented by equation (7) causes the period to vary appreciably with the amplitude or not.

Returning to Fig. 1, let x be the distance from O to P at the time t. Let M' be the mass which by equation (7) corresponds to the elongation x. Let F' represent the force of restitution at the time t. M' will be less than M when x is less than e, and greater than M if x is greater than e. Then from equation (7)

$$F' = M'g = Mg\frac{M'}{M} = Mg\frac{\alpha x + \beta x^2}{\alpha e + \beta e^2}. \tag{11}$$

In equation (3) let $h = \dfrac{\Delta l}{\Delta p} = \dfrac{AT\alpha^2 L}{\omega c}$

$$\Delta l = \Delta x$$
$$\Delta p = M' - M = \alpha(x - e) + \beta(x^2 - e^2).$$

Δx is the increase in length due to the shifting of the position of O by temperature changes in the wire, the temperature changes being caused by the vibration. Then

$$\Delta x = h\,\Delta p = h[\alpha(x - e) + \beta(x^2 - e^2)]. \tag{12}$$

Since equation (7) does not contain this temperature effect, we must add Δx to the x of that equation, and, instead of x, write $x + h[\alpha(x - e) + \beta(x^2 - e^2)]$, Δx is negative when $x < e$. $\Delta x = 0$ when $x = e$. Thus we take the temperature of the wire to be that of the room when $x = e$. We will also suppose the vibrations to be so rapid that there is no time for the wire to receive or to give out heat, but the vibrations take place under adiabatic conditions. Then we have, for the elastic force acting on the mass M, when the distance OP is x

$$F' = Mg\frac{\alpha\{x+h[\alpha(x-e)+\beta(x^2-e^2)]\}+\beta\{x+h[\alpha(x-e)+\beta(x^2-e^2)]\}^2}{\alpha e + \beta e^2}. \tag{13}$$

Since the effect of the weight of the wire itself on the vibrations is equivalent to the effect of one third such weight suspended from the lower end, in what follows, M will denote the suspended mass plus one third the mass of the wire. Except for the copper wire, however, this addition was less than ½ gram, and was neglected.

Let

$$F = F' - Mg. \tag{14}$$

Then if we neglect terms of the order of $h\alpha\beta(x^2 - e^2)$ and $h\alpha\beta x(x - e)$, which are small compared with αx and $\alpha^2 h(x - e)$ we shall have from (13) and (14)

$$F = Mg\frac{\alpha(1 + h\alpha)(x - e) + \beta(x^2 - e^2)}{\alpha e + \beta e^2} = g[\alpha(1 + h\alpha)(x - e) + \beta(x^2 - e^2)]. \tag{15}$$

The external and the internal friction are each assumed proportional to the velocity, and the two terms representing them may be combined thus,

$$\mu\frac{dx}{dt} + \eta\frac{dx}{dt} = \sigma\frac{dx}{dt}, \tag{16}$$

where μ is the external damping coefficient, and η the internal.

As the mass M moves up and down, O (Fig. 1), the point to which the lower end of the wire tends to return, will also oscillate, because of the after-effect. This motion of O will doubtless be somewhat behind the motion of P in phase. The complete vibration requiring only about 1/5 sec. for the lowest frequency observed, there would be only a short time for the wire to recover between vibrations, but immediately after release the motion of O due to the after-effect is comparatively rapid, so it is not at all certain that this motion may be disregarded; in fact, a later discussion will indicate that this is perhaps an important factor in determining the period.

Kohlrausch proposed the formula $-dy/dt = \alpha(y/t)$, or $y = c/t^a$, where y is the deformation due to the after-effect at the time t after the distorting force has ceased to act, and α is a constant. If this were approximately true for small t, we should have a very large dy/dt, but all his observations were taken after 10 sec. and as far as the writer has observed, practically nothing is known of the behavior of the after-effect during the first second after removal of the distorting force. Since an assumption would be worth little without the support of observational data and also since it would doubtless greatly complicate the equation, this effect will for the present be neglected.

The differential equation of motion may then be written

$$\frac{d^2x}{dt^2} + \sigma\frac{dx}{dt} + g\frac{\beta x^2}{M} + \frac{g\alpha(1+h\alpha)}{M}x - \frac{g\alpha^2 he + gM}{M} = 0. \tag{17}$$

If we neglect h and β, then $M = \alpha e$, and (17) becomes

$$\frac{d^2x}{dt^2} + \sigma\frac{dx}{dt} + \frac{g}{e}(x - e) = 0, \tag{18}$$

which is the well-known equation representing damped vibrations, the solution being

$$x = e + Ae^{-\sigma t \div 2}\cos\sqrt{\frac{g}{e} - \frac{\sigma^2}{4}}\cdot t, \tag{19}$$

where the time is counted from the lowest point of the vibration, at which point $x = e + A$, A being the amplitude. The period of this motion is

$$T = \frac{2\pi}{\sqrt{\frac{g}{e} - \frac{\sigma^2}{4}}}. \tag{20}$$

If β is neglected, but h is not, we have

$$T = \frac{2\pi}{\sqrt{\frac{g(1+h\alpha)}{e} - \frac{\sigma^2}{4}}}. \tag{21}$$

From measurements of σ discussed later it will appear that for the largest value of σ found, $\sigma^2/4$ compares with g/e about as 1 with 10^6, showing that under the assumption that the frictional resistance varies as the velocity, the damping has no appreciable effect on the period. Hence, if in (17) we neglect $\sigma(dx/dt)$, we have left

$$\frac{d^2x}{dt^2} + \frac{g}{M}[\beta x^2 + \alpha(1+h\alpha)x - (\alpha^2 he + M)] = 0. \tag{22}$$

This equation represents undamped vibrations, but since we desire only the period that fact is of little importance.

Solution of the Equation.

Let

$$\frac{\beta}{3} = \gamma$$

$$\frac{\alpha(1+h\alpha)}{2} = \delta$$

$$M + \alpha^2 he = \zeta$$

and (22) becomes

$$\frac{d^2x}{dt^2} + \frac{g}{M}(3\gamma x^2 + 2\delta x - \zeta) = 0. \tag{23}$$

By a first integration we obtain

$$\frac{p^2}{2} = -\frac{g}{M}\gamma(x^3 + \frac{\delta}{\gamma}x^2 - \frac{\zeta}{\gamma}x + D), \tag{24}$$

where $p = dx/dt$, and D is the constant of integration.

$$p = \frac{dx}{dt} = \sqrt{\frac{-2g\beta}{3M}}\sqrt{x^3 + \frac{\delta}{\gamma}x^2 - \frac{\zeta}{\gamma}x + D}. \tag{25}$$

D may be determined from the condition, that when $x = e - A$, $p = 0$.

If we let $f(x) = x^3 + (\delta/\gamma)x^2 - (\zeta/\gamma)x + D$ then $f(x) = 0$ when $p = 0$,

$$D = -(e - A)^3 - \frac{\delta}{\gamma}(e - A)^2 + \frac{\zeta}{\gamma}(e - A), \tag{26}$$

and is thus a function of the amplitude.

If $A = e$, which is the greatest amplitude possible, $D = 0$.

This amplitude is always impracticable on account of the tendency of the wire to vibrate transversely when this point is nearly reached.

Equation (25), solved for t, is

$$t = \sqrt{\frac{3M}{-2g\beta}}\int_{x_0}^{x}\frac{dx}{\sqrt{x^3 + \frac{\delta}{\gamma}x^2 - \frac{\zeta}{\gamma}x + D}}. \tag{27}$$

Equation (27) gives the time for any part of the vibration, according to the limits assigned to x. If the extreme positions of P are taken as limits the value of t will be $\frac{1}{2}T$.

To reduce the elliptic integral to the standard form of the first class,

$$F(k, \varphi) = \int_0^{\phi}\frac{d\varphi}{\sqrt{1 - k^2\sin^2\varphi}} = \int_0^{x}\frac{dx}{\sqrt{(1 - x^2)(1 - k^2x^2)}},$$

we must find the roots of $f(x) = 0$.

Since the 4th power of x is lacking, we have at once that one root is ∞, and D has been chosen so that another root is $e - A$. If we substitute the value of D from equation (26) in $f(x) = 0$, and divide by $x - (e - A)$, we obtain

$$x^2 + x\left(e - A + \frac{\delta}{\gamma}\right) + (e - A)^2 + \frac{\delta}{\gamma}(e - A) - \frac{\zeta}{\gamma} = 0, \tag{28}$$

whence

$$x = -\frac{e - A + \frac{\delta}{\gamma}}{2} \pm \frac{1}{2}\sqrt{(e - A)^2 + 2\frac{\delta}{\gamma}(e - A) + \frac{\delta^2}{\gamma^2} - 4(e-A)^2 - 4\frac{\delta}{\gamma}(e - A) + 4\frac{\zeta}{\gamma}}. \tag{29}$$

ζ may be expressed in terms of δ and γ.

$$\begin{aligned}\zeta = M + \alpha^2 he &= \alpha e + \beta e^2 + \alpha^2 he \\ &= \alpha e(1 + h\alpha) + \beta e^2 \\ &= 2\delta e + 3\gamma e^2\end{aligned}$$

$$4\frac{\zeta}{\gamma} = 8\frac{\delta e}{\gamma} + 12e^2. \tag{30}$$

Substitute this value of $4\frac{\zeta}{\gamma}$ in (29) and the equation becomes

$$x = \frac{1}{2}\left(-e + A - \frac{\delta}{\gamma} \pm \sqrt{9e^2 + A^2 + \frac{\delta^2}{\gamma^2} + 6eA + 6\frac{\delta}{\gamma}e + 2\frac{\delta}{\gamma}A - 4A^2}\right). \tag{31}$$

The expression under the radical differs from the square of $3e + A + \delta/\gamma$ only by $4A^2$. For the largest value of A observed

$$\frac{\frac{4A^2}{(\delta)^2}}{(\gamma)^2} = \frac{1}{90000} \text{ for platinum-iridium,}$$

$$= \frac{1}{250{,}000} \text{ for phosphor-bronze,}$$

$$= \frac{1}{640{,}000} \text{ for steel.}$$

For very small amplitudes the ratio is much smaller. Hence $4A^2$ may be neglected and we may write

$$x = \tfrac{1}{2}\left[-e + A - \frac{\delta}{\gamma} \pm \left(3e + A + \frac{\delta}{\gamma}\right)\right]. \tag{32}$$

$$\therefore \left\{\begin{aligned} x_3 &= e + A \\ x_4 &= -2e - \frac{\delta}{\gamma}\end{aligned}\right\} \tag{33}$$

are the remaining two roots of $f(x) = 0$.

It was to be expected that if $e - A$ is one root, $e + A$ is very nearly another, which simply means that in the vibration P moves very nearly as far below the point $x = e$ as it does above that point.

If we assume $e - A$ and $e + A$ to be two of the roots, the third root comes easily from the relation between the roots and the coefficients, for if ω_1, ω_2 and ω_3 are the roots, and

$$\begin{aligned}\omega_1 &= e - A, \\ \omega_2 &= e + A,\end{aligned}$$

then

$$\Sigma\omega = -\frac{\delta}{\gamma}, \tag{34}$$

and

$$\omega_3 = -2e - \frac{\delta}{\gamma}.$$

To obtain the integral in (27) in the standard form we have to transform the roots

$$\infty,\ e - A,\ e + A,\ -\frac{\delta}{\gamma} - 2e$$

respectively into

$$-\frac{1}{k},\ -1,\ 1,\ \frac{1}{k}.$$

The bilinear transformation is

$$x = \frac{py + r}{qy + s}.$$

Determining the constants, we find

$$\begin{aligned} q &= 1, \\ s &= \frac{1}{k}, \\ p &= e + \frac{A}{k}, \\ r &= \frac{e}{k} + A. \end{aligned} \tag{35}$$

If $A = e$, $p = r$.

If $A = o$, $k = o$, and $s = r = \infty$.

$$\int_{x_0}^{x_1} \frac{dx}{\sqrt{f(x)}} = \frac{ps - qr}{\sqrt{M'}} \int_{y_0}^{y_1} \frac{dy}{\sqrt{(1 - y^2)(1 - k^2y^2)}}.$$

$$M' = \frac{\sqrt{b_0}}{k},\ b_0 = q^4 f\left(\frac{p}{q}\right),\ = f(p),\ \text{for } q = 1;$$

$$\therefore \frac{ps - qr}{\sqrt{M'}} = \frac{e + \frac{A}{k} - kA - e}{\sqrt{f(p)}} = \frac{\frac{A}{k} - kA}{\sqrt{f(p)}}, \tag{36}$$

$$k = \frac{1 - \sqrt{a}}{1 + \sqrt{a}},\ \text{where } a = \frac{\omega_4 - \omega_3}{\omega_4 - \omega_2} \cdot \frac{\omega_1 - \omega_2}{\omega_1 - \omega_3}. \tag{37}$$

The three finite roots are all positive, and the graph of the function appears somewhat as in Fig. 10. The integration takes place from $\omega_2 = e - A$ to $\omega_3 = e + A$, and is therefore real.

D is negative except when $A = e$ in which case it is zero.

$$a = \frac{\frac{\delta}{\gamma} + 3e + A}{\frac{\delta}{\gamma} + 3e - A}. \tag{38}$$

Let $c = -(\delta/\gamma) - 3e$, a positive quantity. Then

$$k = \frac{1 - \sqrt{\frac{c-A}{c+A}}}{1 + \sqrt{\frac{c-A}{c+A}}} = \frac{c - \sqrt{c^2 - A^2}}{A}. \tag{39}$$

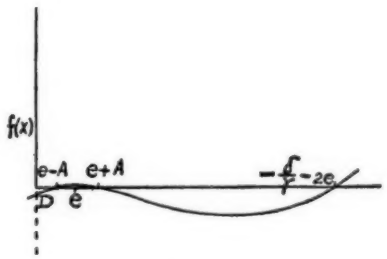

Fig. 10.

When $A = 0$, $k = 0$, and the last member of equations (36) becomes indeterminate. To evaluate it, from (39)

$$\frac{A}{k} = \frac{A}{c - \sqrt{c^2 - A^2}} = c + \sqrt{c^2 - A^2}, \tag{40}$$

hence,

$$\lim_{A=0} \frac{A}{k} = 2c, \tag{41}$$

also from (39),

$$Ak = c - \sqrt{c^2 - A^2}; \tag{42}$$

$$\therefore \frac{A}{k} - kA = 2\sqrt{c^2 - A^2} \tag{43}$$

and

$$\frac{ps - qr}{\sqrt{M'}} = \frac{2\sqrt{c^2 - A^2}}{\sqrt{f(p)}}; \tag{44}$$

$$\therefore t = \sqrt{\frac{3M}{-2g\beta}} \int_{x_0}^{x_1} \frac{dx}{\sqrt{f(x)}} = \frac{2\sqrt{c^2 - A^2}}{\sqrt{f(p)}} \times \sqrt{\frac{3M}{-2g\beta}} \int_{y_0}^{y_1} \frac{dy}{\sqrt{(1-y^2)(1-k^2y^2)}}. \tag{45}$$

When $x_0 = e - A$, $y_0 = -1$; when $x_1 = e + A$, $y_1 = 1$. Integrating between these limits we have $\frac{1}{2}T$. Hence the period is given by

$$T = \frac{8\sqrt{c^2 - A^2}}{\sqrt{f(p)}} \sqrt{\frac{3M}{-2g\beta}} K, \tag{46}$$

where K is the value of the complete elliptic integral

$$\int_0^1 \frac{dy}{\sqrt{(1-y^2)(1-k^2y^2)}}.$$

k was found to be very small for all values of A really obtained, and it approaches zero with A. Thus if $A = 0.5$ cm. for platinum-iridium, $\sin^{-1} k = 22'$. If $A = 1.0$ cm. for steel $\sin^{-1} k = 7'$. But the largest value of A given in Figs. 11 and 12 is about 0.1 cm. This gives for platinum-iridium $\varphi = \sin^{-1} k = 4'\ 21''$ and for steel $\varphi = 0'\ 42''$. For phosphor-bronze φ is slightly larger than for steel. If $k = 0$,

$$\int_0^1 \frac{dy}{\sqrt{(1-y^2)(1-k^2y^2)}} = \int_0^1 \frac{dy}{\sqrt{1-y^2}} = \frac{\pi}{2} = 1.5708.$$

If $\sin^{-1} k = 1°$,

$$\int_0^1 \frac{dy}{\sqrt{(1-y^2)(1-k^2y^2)}} = 1.5709.$$

Since k is in no case larger than sin 30′, and for ordinary amplitudes is much smaller, we may use $K = \pi/2$ for all the wires and all amplitudes.

The effect of varying amplitude on the other factors in eq. (46) will be shown after α and β have been determined.

Simplification of the Equation (46) for Small Amplitudes.

When $A = 0$, (46) becomes

$$T = 2\pi \frac{2c}{\sqrt{f(p)}} \sqrt{\frac{3M}{-2g\beta}}. \tag{47}$$

This may be further simplified,

$$D = -f(e), \text{ when } A = 0, \tag{48}$$

$$\frac{\zeta}{\gamma} = 2\frac{\delta}{\gamma}e + 3e^2,$$

$$f(p) = p^3 - e^3 + \frac{\delta}{\gamma}(p^2 - e^2) - \frac{\zeta}{\gamma}(p - e)$$

$$= (p-e)\left[p^2 + pe + e^2 + \frac{\delta}{\gamma}p + \frac{\delta}{\gamma}e - 2\frac{\delta}{\gamma}e - 3e^2\right]$$

$$= (p-e)\left[p^2 + pe - 2e^2 + \frac{\delta}{\gamma}p - \frac{\delta}{\gamma}e\right],$$

$$f(p) = (p-e)^2\left(p + 2e + \frac{\delta}{\gamma}\right). \tag{49}$$

From equation (35)

$$p = e + \frac{A}{k} = e + 2c, \text{ when } A = 0; \tag{50}$$

$$\therefore f(p) = (2c)^2(e + 2c - e - c) = 4c^3. \tag{51}$$

Hence, from (47),

$$T = 2\pi \frac{1}{\sqrt{c}} \sqrt{\frac{3M}{-2g\beta}} = 2\pi \sqrt{\frac{3M}{-2g\beta(-\frac{\delta}{\gamma} - 3e)}},$$

$$T = 2\pi \sqrt{\frac{M}{g[\alpha(1 + h\alpha) + 2\beta e]}}, \tag{52}$$

$$n = \frac{1}{2\pi} \sqrt{\frac{g[\alpha(1 + h\alpha) + 2\beta e]}{M}}. \tag{53}$$

This is the final form for the expression of the frequency for vibrations of small amplitude, in which the motion is along the curve represented by the equation $M = \alpha e + \beta e^2$, except for the small heating effect indicated by the term containing h. The effect of larger amplitudes as shown by (46) will be mentioned later.

Omitting the term $h\alpha$, (53) is the same as (10). If the wire obeyed Hooke's Law, we would have only to set $\beta = 0$, and (53) reduces to

$$n = \frac{1}{2\pi} \sqrt{\frac{g\alpha(1 + h\alpha)}{M}} = \frac{1}{2\pi} \sqrt{\frac{g(1 + h\alpha)}{e}}, \tag{54}$$

which is equivalent to (21) except for the damping term.

Computation of Constants for the Equation $M = \alpha e + \beta e^2$.

The next step is to find the constants α and β of the equation $M = \alpha e + \beta e^2$, which will best satisfy the observed values of the frequency as given by equation (53).

We have from (53)

$$Mn^2 = \frac{1}{4\pi^2}[g\alpha(1 + h\alpha) + 2\beta e].$$

Since $2\beta e$ is small compared with $g\alpha(1 + h\alpha)$ it will be sufficient to write M/α in place of e; for $e = M/\alpha - \beta e^2/\alpha$, and the maximum value of $\beta e^2/\alpha$ is about one per cent. of M/α. This will influence the determination of β only, and one per cent. in the value of β is negligible. Making this substitution for e in (53), the equation becomes

$$Mn^2 = \frac{1}{4\pi^2}\left[g\alpha(1 + h\alpha) + \frac{2\beta}{\alpha} M\right]. \tag{55}$$

If we plot Mn^2 as a function of M, we obtain for steel, phosphor-bronze, and platinum-iridium the results shown in Fig. 11. The slope of each of these lines is $\beta/2\pi^2\alpha$ for that wire, and the intercept on the Mn^2 axis is $g\alpha(1 + h\alpha)/4\pi^2$. $h\alpha$ is of the order 0.001 for all the three wires to which the equation applies, hence for $h\alpha$ we may use the value obtained by neglecting h and β altogether. The values of $1 + h\alpha$ are

Copper	1.0021
Steel	1.0013
Phosphor-bronze	1.0015
Platinum-iridium	1.0011

The heating effect has therefore little influence on the frequency.

It will be noticed on the steel and platinum-iridium plots that when M is small Mn^2 is less than it should be. This may be explained by supposing that the wire obeys Hooke's Law for small elongations. Also, as

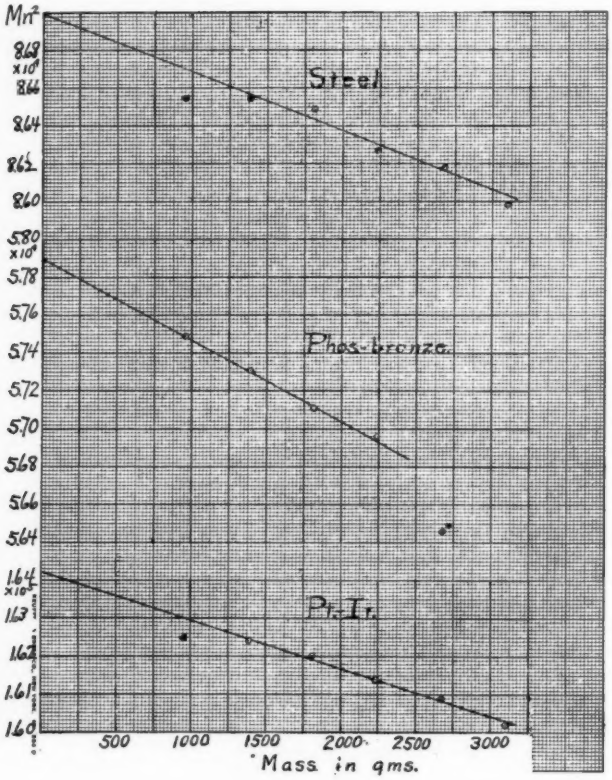

Fig. 11.

will appear later in comparing the results for direct elongation, the wires seem not to be entirely straightened out until the second mass is reached, in spite of the precautions, already noted, which were taken to straighten them. The phosphor-bronze wire does not exhibit this deficiency in Mn^2 because the elongation of that wire was relatively greater, and the first weight took the wire beyond the region where the elongation is proportional to the load. The largest weight used was too great for the phosphor-bronze wire, and although no permanent elongation could be observed when this mass was removed, yet it is plainly evident from the

small value of Mn^2 for this mass that the rate of elongation of the wire at this point was greater than that which is given by $M = \alpha e + \beta e^2$.

For the copper wire, Mn^2 is a constant within the range of error, but it is to be noted that the greatest elongation is only about the same as the range for which Hooke's Law holds in the case of the steel and platinum-iridium wires. If there is any range for which equation (7) represents the behavior of the copper wire, that range is very limited, for the product of Mn^2 is a constant as far as observed, and the wire could sustain only a little more than the heaviest weight without passing the elastic limit.

From Fig. 11 the following equations were obtained.

$$\text{Steel: } M = 3{,}498\ e - 22.0\ e^2. \tag{56}$$

$$\text{Phosphor-bronze: } M = 2{,}327\ e - 19.6\ e^2. \tag{57}$$

$$\text{Platinum-iridium: } M = 6{,}602\ e - 166\ e^2. \tag{58}$$

To show with what degree of accuracy these represent the observed frequencies, we write eq. (53) in the form

$$n = \frac{1}{2\pi}\sqrt{\frac{g\alpha(1 + h\alpha)}{M} + 2\frac{\beta}{\alpha}g}, \tag{59}$$

and compute n for each mass, comparing the observed with the computed values.

TABLE VIII.

Steel.				Phosphor-bronze.				Platinum-iridium.			
M Gm.	n_0.	n_c.	n_0-n_c.	M Gm.	n_0.	n_c.	n_0-n_c.	M Gm.	n_0.	n_c.	n_0-n_c.
954	9.525	9.531	−0.006	953	7.767	7.766	+0.001	953	13.058	13.076	−0.018
1,384	7.908	7.908	.000	1,383	6.437	6.436	+0.001	1,383	10.837	10.836	+0.001
1,815	6.903	6.900	+ .003	1,814	5.611	5.611	.000	1,814	9.450	9.449	+0.001
2,245	6.199	6.199	.000	2,244	5.038	5.037	+ .001	2,244	8.481	8.479	+0.002
2,676	5.675	5.673	+ .002	2,675	4.594	4.606	− .012	2,675	7.755	7.753	+0.002
3,098	5.268	5.269	−0.001					3;097	7.193	7.193	0.000

n_0 = observed value of the frequency for infinitely small vibrations, taken from Fig. 9 for platinum-iridium.

n_c = frequency computed from equation (59).

The above residuals show that the frequency is very well represented by equation (59) except for the first masses of steel and platinum-iridium and the last of phosphor-bronze. For the smallest masses it is not certain that the wires were fully straightened, but on examining the results for all the wires including the copper we must conclude that for small distortions, the elongation is proportional to the distorting force, beyond that there is a region where the wire very closely obeys the law expressed by equation (7), then the elongation increases more rapidly

TABLE IX.

Copper.					Phosphor-bronze.					Platinum-iridium.				
M[1] Gm.	e_0 Cm.	n_0.	n_c.	n_0-n_c.	M gm.	e_0 Cm.	n_0.	n_c.	n_0-n_c.	M Gm.	e_0 Cm.	n_0.	n_c.	n_0-n_c.
956	0.0822	17.716	17.397	+0.319	953	0.4138	7.767	7.752	+0.015	953	0.1461	13.058	13.044	+0.014
1,386	0.1184	14.738	14.496	.242	1,383	.6023	6.437	6.426	+0.011	1,383	.2119	10.837	10.831	+0.006
1,817	0.1563	12.871	12.617	.254	1,814	.7912	5.611	5.606	+0.005	1,814	.2780	9.450	9.456	−0.006
2,247	0.1945	11.578	11.310	.268	2,244	.9816	5.038	5.033	+0.005	2,244	.3448	8.481	8.491	−0.010
2,678	0.2320	10.602	10.356	.246	2,675	1.1742	4.594	4.602	−0.008	2,675	.4137	7.755	7.752	+0.003
3,100	0.2695	9.845	9.609	+0.236						3,097	.4812	7.193	7.187	+0.006

[1] M does not include the mass of the scale-pan, which was 223 gm. e_0 is the change in elongation due to M. For the vibrations, M is the tota mass suspended.

even than this formula demands, and finally the strain becomes so great that the wire gives way. The weaker parts of the wire pass through these stages in advance of the stronger parts, and their cross-section being lessened, they elongate more and more rapidly, until rupture takes place at one of these points.

Computation of Frequency for Wires Obeying Hooke's Law.

Formula (21) gives the period if the wire obeys Hooke's Law and if we measure the elongation directly. For small elongations we might expect this formula to give a period agreeing well with the observed, but it does not. In all cases the e observed directly and substituted in (21) gives too small values for n, indicating that the observed e is too large.

Table IX gives the values of n_e computed from formula (21), using the values of e observed directly. The masses are the same as those used in the vibrations, except for steel. For this reason steel is given separately in Table X.

At first sight it might be supposed that these results for phosphor, bronze and platinum-iridium are satisfactory, with the exception-perhaps, of the smallest mass; and that formula (21) gives the period more closely than (59): but it must be remembered that formula (21) assumes that $e \propto M$. Therefore we have no right to substitute in it the values of e_0 in Table IX, because the ratio e_0/M is not constant, and we

TABLE X.

Steel.

M Gm.	$\frac{e}{M}$.	M Gm.	e Cm.	n_0.	n_e.	$n_0 - n_e$.
1,000	2.874×10^{-4}	954	0.2754	9.525	9.501	+0.024
1,500	2.873	1,384	.3996	7.908	7.888	+0.020
2,000	2.883	1,815	.5240	6.903	6.888	+ .015
2,500	2.885	2,245	.6481	6.199	6.193	+ .006
3,000	2.892	2,676	.7726	5.675	5.673	+ .002
Mean...	2.887×10^{-4}	3,098	.8944	5.268	5.272	− .004

need to use a different ratio of e_0/M for each one, which is contrary to the hypothesis on which (21) is derived.

Columns 3 and 4, Table X, give the vibrating masses and the corresponding elongations, which were computed from the mean e/M by multiplying by the mass used in vibration. n_e is the frequency from

$$n = \frac{1}{2\pi}\sqrt{\frac{g(1 + h\alpha)}{e}},$$

using the e of column 4. Table X. shows, as the values of Mn^2 of Table VII. also do, that no constant value of M/e will satisfy the observed frequencies.

Comparison of e from Static and Dynamic Observations.

Finally the elongations corresponding to the observed frequencies were computed for all the wires. For the copper wire the formula, $e = g(1 + h\alpha)/4\pi^2 n^2$ was used. For the others formulæ (56), (57), and (58) were inverted, giving e in the form of equation (8).

$$\text{Steel: } e = 2.859 \times 10^{-4} M + 5.1 \times 10^{-10} M^2. \quad (60)$$

$$\text{Phosphor-bronze: } e = 4.297 \times 10^{-4} M + 1.6 \times 10^{-9} M^2. \quad (61)$$

$$\text{Platinum-iridium: } e = 1.515 \times 10^{-4} M + 5.9 \times 10^{-10} M^2. \quad (62)$$

For these three wires e_c was computed for the given load plus the pan, then for the pan alone, and the difference compared with the elongation, e_0, observed directly. In the case of copper since the product of Mn^2 was a constant no computation was made for the initial load.

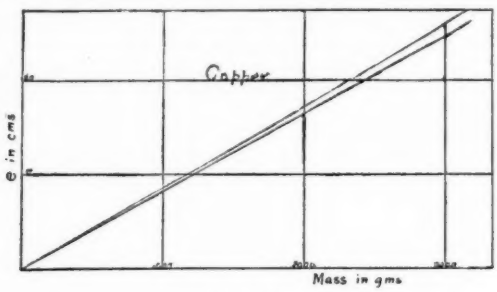

Fig. 12.

Fig. 12 shows e_0 and e_c for copper as ordinates with M on the axis of abscissas. e_c and M give a straight line, while e_0 and M do not. For the other wires neither curve is a straight line, and they lie so near together that they have not been plotted. $e_0 - e_c$ is always positive, however, for all the wires, and increases with M.

V. Discussion of Results.

The dynamic modulus of metals has been found by other methods to be greater than the static modulus. The same is true for these longitudinal vibrations, as shown in the last paragraph. No attempt has been made to determine the absolute value of either modulus, it being outside the purpose of this investigation.

If we consider the modulus to be proportional to dM/de, we have from equation (7) $dM/de = \alpha + 2e\beta$, which may easily be computed by equations (56), (57) and (58). For copper, since $\beta = 0$, we have $M/e = 4\pi^2 Mn^2/g(1 + h\alpha)$.

Possible After-effect in All the Wires.

A large after-effect is shown in Fig. 5 for copper, and some is also exhibited by similar curves for steel. Since the recovery is rapid at first, it seems quite possible that the point of instantaneous recovery may be far beyond that indicated by the dotted continuation of the curves. The greater part of the recovery may be during the first second or two following release, or even while the load is being removed. The curves for copper unquestionably show that it is rapid at first, for very little of the effect there indicated comes from the heating of the wire. For phosphor-bronze and platinum-iridium there is *apparently* no after-effect, but the recovery may be so rapid that it is complete before an observation can be taken. This theory would explain the discrepancy between the observed elongation and that computed from the results of vibrations, which is equivalent to saying it would account for the difference between the static and dynamic moduli.

We may suppose, then, that the observed values of the elongation are all too large. The deformation due to the after-effect is much larger than would be supposed from usual observation, for it decreases very rapidly when the load is first removed, and by the time an observation can be made a great part of it has disappeared. Thus there may be considerable after-effect in wires which apparently show none; the wire may have entirely recovered by the time the first observation can be made.

Influence of the After-effect on the Period.

Considering again Fig. 1, as the point P oscillates, O will also oscillate with a certain amplitude depending on the period, the distance OP, and the material of the wire, as well as its condition as to hardness, temperature, etc. The phase difference between the motions of O and P will depend on the period and the manner in which the wire recovers after removal of the load. This last is an important element, and also an uncertain one. If the phase difference were zero, then P and O would both move downward through their mean positions at the same instant. In the half of the period when P is below its mean position, O would also be below its mean position. As a result the force of restitution would be less than if O remained at rest, and the period would be increased. When P and O were above the mean positions there would be a greater elastic tension on account of the displacement of O, and this would also increase the period, so that we have a decrease in frequency due to the motion of O. It may be noted that this is just opposite to the effect of the motion of O due to the heating and cooling of the wire. (Comp. the term $h\alpha$ in equation (53).)

But we must suppose that O is somewhat behind P in phase. Then when the two points are on opposite sides of their respective positions of rest the frequency would tend to increase, when they are on the same side it would tend to decrease. Unless the motion of O became 90° or more behind that of P, the frequency would probably be decreased by this motion of O.

That this does not account for the variation in Mn^2 for steel, phosphor-bronze, and platinum-iridium, appears from the fact that Mn^2 did not vary appreciably in the case of copper, which is just the one in which the after-effect was greatest. It is true that the recovery of copper is not so rapid as that of the other three metals, and hence the amplitude of O may not be as great, but we should expect at least a noticeable variation in Mn^2.

VI. The Damping of Longitudinal Vibrations.

Apparatus.

The amplitude of the vibrations when the wire is only about two meters long cannot be very great; consequently the accurate measurement of the damping becomes a matter of some difficulty. The following method has yielded fairly satisfactory results.

A fiber of glass was attached to the wire, and allowed to project about 1 cm. at right angles to it. The fiber pointed towards a specially constructed camera. The lens, which was one used in an ordinary galvanometer telescope, and had a focal length of about 15 cm., was stopped down until the aperture was only about 5 mm. At the back of the camera was fixed a drum, 40 cm. in circumference, rotating about a vertical axis. The axis was a screw, so that as the drum revolved, it moved parallel to the axis. A small drop of mercury was placed on the end of the glass strip next to the camera, the drop was brightly illuminated by the light from an electric arc, and the camera was so placed that the image of the drop fell on the drum. Owing to the great curvature of the small mercury drop, a sharp spot of light was obtained. A rapid photographic film was attached to the drum, and rotated as the wire vibrated, thus obtaining a record of the vibration. The mercury drop could not be placed directly on the wire, because of the fogging of the film by light reflected from the wire. The glass fiber was colored red, so that any light reflected therefrom would not affect the record. Fig. 13 shows a section of a record for steel.

Since the purpose was only to measure the amplitude, uniformity of motion of the drum was unnecessary. It was therefore turned by hand, a handle outside the camera box serving that purpose. The mass was set vibrating and the drum started. Then the current driving the wire

Fig. 13. Steel.

was shut off and at the same time the drum was stopped for an instant. The several vibrations thus heaped up together indicated where the damping began.

The measurements of the amplitude were made on a measuring engine of the Detroit Observatory, the plate-holder of the engine being mounted to revolve about an axis parallel to the axis of the microscope. This permitted an easy adjustment of the cross hair tangent to two adjacent waves. For all the wires except copper, 1,000 vibrations could easily be measured. Beyond that number they were so small that a relatively large error in the logarithm of the amplitude was introduced. The lens was so placed that the amplitude on the film was about equal to the actual amplitude of the point on the wire. The motion might easily

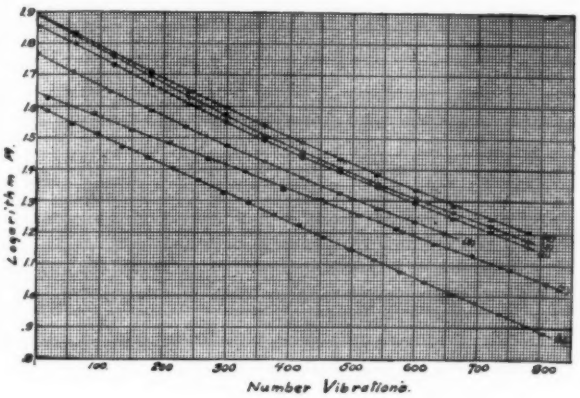

Fig. 14.

Decrement curves for steel.

have been magnified, but what was gained in this way was lost in definition of the trace. The largest double amplitude measured was about 4 mm. on the film, the smallest about 0.3 mm. Ordinarily the measurements ran from 3 mm. to 0.5 mm. The measuring engine read directly to 0.005 mm.

Two records were made for each mass suspended from the copper wire, because the rapid damping allowed room for two on the film. For the other metals the damping was so small that only one record was made on each film. Every 30th vibration was measured and the logarithms of these were averaged by pairs, so that the plots show only the logarithms for every 60th vibration. Figs. 14 and 15 represent graphically the decrements for the masses given in Table XI. in order of increasing magnitude. Similar curves were drawn for copper, and the logarithmic decrements of Table XI. were obtained from these curves.

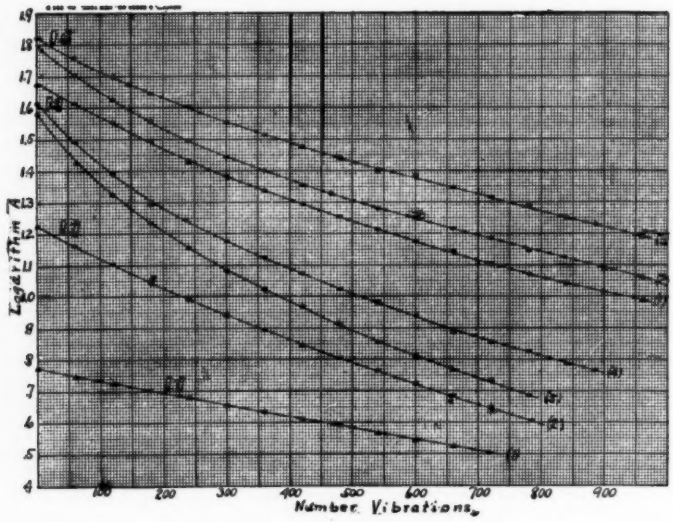

Fig. 15.

Decrement curves, ⊙, for platinum-iridium; △, for phosphor bronze. The scale is the same for all; but to avoid confusion the origin of ordinates has been shifted up or down for certain curves, as indicated by the logarithms in brackets.

Variation of the Logarithmic Decrement.

It was found by Schmidt[1] for torsional, and later also by Voight[2] for flexural and torsional vibrations, that the logarithmic decrement varied with the amplitude. They expressed the relation in the form

$$\lambda = \lambda_0 + bA^2,$$

where λ_0 is the logarithmic decrement for very small vibrations.

The values of Table XI. for copper are plotted in Fig. 16. Although the rate of increase is different for different masses, the increase is nearly proportional to the increase in amplitude[3] in all cases, and

$$\lambda = \lambda_0 + bA$$

represents the relation. The second mass gave an abnormally large decrement for the larger amplitudes, as seen from Table XI. There is no apparent reason for this. Both records taken of this mass show the same peculiarity.

While with the other wires, the logarithmic decrement for a rough approximation, increased in proportion to the increase of amplitude, the increase was much less than with copper, and less regular. The logarith-

[1] Loc. cit.

[2] Loc. cit.

[3] Bouasse and Carriere found $\lambda = bA$ for torsional vibrations of copper. (See Ann. de Chim. et de Physique, 8me Serie, t. 14, p. 208.)

TABLE XI.

Log. Decr.—Amplitude. Copper.

λ in common logarithms.

Mass. Gm.	Log. A.		1.8	1.7	1.6	1.5	1.4	1.3°	1.2	1.1	1.0	.9	.8	Temp. C.	n_0
	A_0	A	6.31	5.01	3.98	3.16	2.51	1.99	1.58	1.26	1.00	.79	.63		
956	2.30	λ						.00274	247	224	206	179		21°.9	17.716
	1.55									.00222	198	181	169		
1,386	2.75							.00475	384	305	259	226		24 .2	14.738
	2.85						.00571	494	377	274	232				
1,817	5.95			.00559	470	394	362	308	277	256	226	219		24 .3	12.871
	6.30			.00546	475	410	367	309	277	240	225				
2,247	3.80					.00354	320	292	252	228	208	203		22 .9	11.578
	3.95					.00342	329	283	254	219	205	198			
2,678	5.20				.00453	404	386	348	323	293	263	250		24 .2	10.602
	5.20				.00454	420	374	339	294	280	264	251			
3,100	4.05					.00408	364	320	296	270	239	221		24 .2	9.845
	6.55			.00562	490	436	370	325	292	259	242	230			

Steel.

M Gm.	A_0	A	6.31	5.01	3.98	3.16	2.51	1.99	1.58	1.26	1.00	Temp. C.	n_0
954	4.00	λ				.00091	90	90	90	88	85	23°.6	9.525
1,384	4.35				.00079	77	74	74	72	70		23 .6	7.908
1,815	6.15			.00103	99	91	84	79				23 .8	6.903
2,246	8.15		.00102	99	92	89	84	80				22 .7	6.199
2,676	8.25		.00114	107	100	92	85	78				22 .5	5.675
3,098	7.70			.00110	107	98	91	85	80	76		22 .3	5.268

TABLE XI.—*Continued.*

Phosphor-bronze.

M Gm.	A_0	A	6.31	5.01	3.98	3.16	2.51	1.99	1.58	1.26	1.00	Temp. C.	n_0
1,383	6.05			.00040	37							21 .7	6.437
2,244	5.60				.00100	87	78	69	64			21 .8	5.038
					Platinum-iridium.								
953	4.95				.00107	101	82	71	63	58	45	23 .7	13.058
1,383	6.50			.00153	115	93	72	61	57	49		23 .9	10.837
1,814	4.35				.00108	92	77	65	54	52	50	24 .3	9.450
2,240	7.15			.00188	156	126	102	85	76	70	57	24 .2	8.481
2,675	6.70			.00245	190	157	139	115	96	89	75	24 .2	7.755

The column A_0 contains the largest amplitude measured on each film. The last two columns give the temperatures inside the box when the photographs were taken, and the frequencies for small amplitudes.

mic decrement for steel, especially for the smaller masses, is very nearly constant.

For phosphor-bronze only two masses were used and there is a remarkable difference in the decrement in these two cases. The decrement for the small mass, 1,383 gm., is only about one third that for the larger mass, 2,244 gm., and that for the smaller mass is very nearly constant. In 700 vibrations this amplitude decreased from 6.04 to 3.27, while for the larger mass in the same number of vibrations it decreased from 5.56 to 1.48. For the other metals there is no apparent dependence of λ

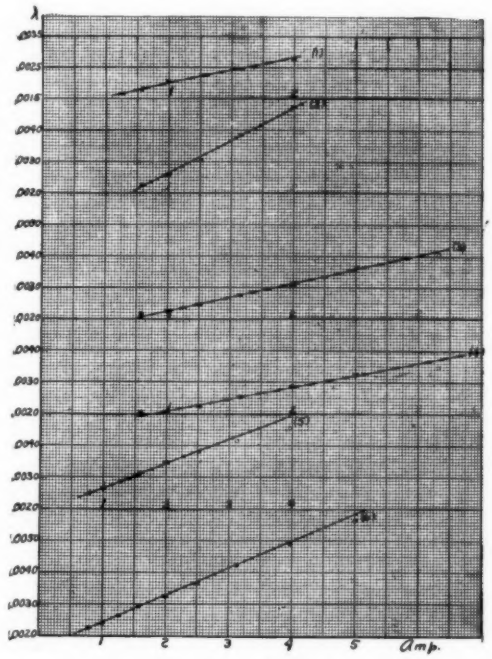

Fig. 16. Copper.

on the period. If we produce the straight lines of Fig. 16, backward to the axis we get for λ_0, omitting No. 2

Mass.	λ_0.
(1)	.00121
(2)	———
(3)	.00136
(4)	.00130
(5)	.00186
(6)	.00160

With the largest mass suspended from the copper wire a single test was made of the effect of starting with a larger amplitude. In the case of the greater initial amplitude the decrement is slightly higher for large

amplitudes, but as the amplitudes decrease the decrements in the two cases approach equality.

The temperatures were nearly the same in all cases, so little variation can be expected from this source.

It would be very desirable to test more fully a single wire, carrying a constant load, to determine the effect of change of temperature, and previous treatment of the wire, as well as the variations of other conditions for which there was not opportunity in this limited investigation.

Cause of Damping.

If the damping were caused by internal friction of the particles, and the friction were assumed proportional to the velocity, then we should have $\lambda \propto \frac{T}{M} \propto n$, or λ/n = a const. That this is not fulfilled at all by any of the wires is plainly seen by reference to Table XI., and by comparing λ for equal amplitudes and different frequencies. The phosphor-bronze shows a marked variation in the other direction; λ decreases with increase of frequency.

In Boltzmann's theory the decrement is independent of the period, and this seems to be nearer the truth, for longitudinal vibrations. As already stated in the introduction, Voight found for torsional and flexural vibrations that some metals showed, as phosphor-bronze has done in the present instance, an increase of λ with decrease of n. A definite conclusion can not be reached for phosphor-bronze, however, since but two frequencies were used.

Correction for Air Damping.

A correction for the damping due to the friction of the air was made as follows. The cylindrical shell shown in Fig. 6 with the rod attached as when used for vibrations, was suspended in a horizontal position by a bifilar thread, so as to swing in a direction parallel to the axis, thus exposing the same surface and giving it the same motion through the air. The wire in longitudinal vibration presented only a small surface compared with the shell, and its effect would be nearly compensated by the friction of the bifilar threads mentioned above. The logarithmic decrement of this pendulum system was then determined for the different masses.

From equation (16) $\mu = 2\lambda_0/T_0$, where λ_0 and T_0 are respectively the logarithmic decrement, and period of the pendulum vibrations.

$\eta = 2(\lambda/T - \lambda_0/T_0) = 2\lambda_m/T_m$, where λ_m is the logarithmic decrement

due to the internal damping alone. T_m is the period as it would be if there were no air damping, and is practically identical with T.

$$\therefore \lambda_m = \lambda - \lambda_0 \frac{T}{T_0}.$$

Thus $\lambda_0(T/T_0)$ is the correction to be subtracted from the measured value of λ to obtain the decrement due to the metal alone.

For small amplitudes the following were observed.

Mass.	λ_0.	T sec.
2,244	0.00027	3.44
3,097	0.00027	3.44

The greatest correction to be applied is for the smallest value of n. The smallest value of n was for phosphor-bronze, where for $M = 2{,}244$ gm., $n = 5.04 \frac{\text{vibs.}}{\text{sec.}}$

There was no mass of 3,097 gm. used with phosphor-bronze, but for steel

$$M = 3{,}097 \text{ gm.}, \quad n = 5.27$$
$$M = 2{,}244 \text{ gm.}, \quad n = 6.20$$

Thus we have for phosphor-bronze,

M gm.	$\frac{\lambda_0 T}{T_0}$.
2,244	0.000019

for steel,

2,244	0.000015
3,097	0.000016

Therefore the maximum correction to be applied to λ given in Table XI. is 0.00002. For the copper it would be about half this value, and hence, when compared with the decrement for internal damping, is small enough to be neglected.

The Platinum-Iridium Wire.

The platinum-iridium wire was chosen because of the peculiar behavior of such wires noticed by Guthe,[1] and later investigated further by the same author and Sieg.[2] They found an excessive damping of torsional vibrations for platinum-iridium wires containing 40 per cent. of iridium, which is the same composition as that of the wire used in the present instance. It was accordingly expected that the damping of the longitudinal vibrations would also be large. The result was a decrement comparable to that of steel and phosphor-bronze, and far below that of the copper. When the wire was later suspended and vibrated torsionally, it was found that neither did this wire exhibit unusually large damping of

[1] Proc. Iowa Ac. Sci., 15, p. 147, 1908.

[2] PHYS. REV., Vol. XXX., No. 4, 1910.

torsional vibrations. The highest decrement observed for the torsional vibrations was 0.00270, when the initial amplitude was $82\frac{1}{2}°$. The decrement decreased considerably with the amplitude. Since the length of the wire under discussion was 145 cm. and the one used by Guthe and Sieg was 40 cm., an angle of 825° is about double the twist per unit length for which they observed a logarithmic decrement some $2\frac{1}{2}$ times as large. The diameter of the wire used here was 0.206 mm., as compared with 0.194 mm. for theirs. Their wire showed a difference in period of 2 per cent. between large and small amplitudes. The period of this wire was found to be 18.069 sec. for 11° amp., 18.133 sec. for 550° amp., a difference of only $\frac{1}{3}$ per cent. There is, therefore, evidently a difference in the composition of the two wires, or else previous treatment is very influential in determining their behavior.

Variation of Period with Amplitude.

The effect of damping on the period is given by

$$T = \frac{2\pi}{\sqrt{\frac{g}{e} - \frac{\sigma^2}{4}}},$$

where $\sigma = 2\lambda/T$.

For copper, which exhibited the greatest damping, let us take $\lambda = 0.010$ in natural logarithms, which corresponds to the highest λ computed in common logarithms. Also take $T = 0.07$ sec., which is approximately the period for $M = 1{,}386$ gm. Then

$$\sigma = \frac{.02}{.07} = 0.29, \quad \frac{\sigma^2}{4} = 0.020,$$

$$\frac{g}{e} = \frac{980}{.12} = 8170.$$

Hence $\sigma^2/4$ compares with g/e about as 1 with 400,000. The observed variation of T was $\frac{1}{3}$ per cent., hence the variation was not caused by the change in λ, provided σ enters into the equation for the period as shown above. The fact that λ varies so greatly with the amplitude shows however, that the internal friction, whatever may be its nature, is *not* proportional to the velocity.

The equation (46) is an expression for the period which involves the amplitude. The largest value of A observed in timing was 0.1 cm. Substituting this in equation (46) we find that for platinum-iridium, $\sqrt{c^2 - A^2}$ differs from c by 1 part in 700,000, and in the denominator, $\sqrt{f(p)}$ differs, when $A = 0$, and when $A = 0.1$ cm. by only 1 part in 3,000,000. It has already been stated that $K[= F(k, \pi/2)]$ is not perceptibly affected by

changing A within the limits of the experiment; hence the vibration along the curve represented by (60), (61), or (62) will not account for the variation of T. This also appears by considering that for copper the dynamic modulus was a constant, yet the greatest variation of period with amplitude was observed here.

By comparing elongations from static and dynamic observations as in Fig. 12 it is found that the differences $e_0 - e_c$ per unit length, in the order of magnitude beginning with the largest, place the metals in the following order: Copper, platinum-iridium, phosphor-bronze, steel.

This is the order of variation of period with amplitude; the copper showed the greatest variation, the platinum-iridium considerable, the phosphor-bronze very little and none was detected for steel.

The position of the wires with regard to the magnitude of λ is uncertain for all except copper, which again undoubtedly stands at the head.

The indication is that the wires having the largest after-effect, have also the greatest variation of period with amplitude, and the largest decrement, although it is to be noted that *rapidity of recovery* from after-effect is of as much importance in affecting the period as the total magnitude of the after-effect.

VII. Summary.

1. As a result of the measurement of the frequency of longitudinal vibrations of wires, carrying various loads, it appears in Section IV. that for small loads, the modulus is a constant; for greater loads the modulus decreases with increasing load, although the wire is still far within the elastic limit.

2. These two regions are of different relative extent in different wires. A soft copper wire showed a constant modulus until the elastic limit was very nearly reached (see Table VII.). Steel, phosphor-bronze and platinum-iridium, showed a relatively large range of elongation where the frequencies for small amplitudes are satisfied by an equation of the form

$$n = \frac{1}{2\pi}\sqrt{\frac{g[\alpha(1 + h\alpha) + 2\beta e]}{M}} \text{ (see Table VIII.),}$$

where, for a given wire α and β are constants, e is the elongation for the mass M, and e and M are connected by the relation $M = \alpha e + \beta e^2$. h is a small constant depending on temperature changes in the wire as the wire vibrates.

3. The modulus determined dynamically is, as usual, larger than that measured under static conditions (see Fig. 12).

4. This difference in moduli may be explained as the result of a large after-effect, the greater part of which disappears very rapidly after the removal of load (see Sec. V.).

5. The frequency decreases with increase of amplitude, in the case of copper and platinum-iridium. The relation may be represented by $n = n_0 - bA$ where b is a positive quantity. b is greater for large frequencies than for small (Tables V. and VI.). The variation for phosphor-bronze was exceedingly small, and for steel piano-wire n was evidently independent of the amplitude.

6. The wire which showed the greatest variation of period with amplitude also showed the greatest damping and the greatest after-effect.

7. The logarithmic decrement was measured and found to vary with the amplitude (Sec. VI.). For copper, at least, the increase of λ was very nearly proportional to increase of amplitude (Fig. 16). The logarithmic decrement does not vary with different frequencies in such a manner as to indicate that the damping is due to internal friction.

The writer is under obligation to Professor Guthe for proposing the investigation, and for making valuable suggestions during the progress of the work, to Professor Randall for advice in regard to some of the experimental part, and to Professor Curtiss for the use of a measuring engine of the Detroit Observatory.

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UNIVERSITY OF MICHIGAN,
March, 1914.

A COMPARISON OF ACTUAL AND BLACK-BODY TEMPERATURES

BY CHARLES C. BIDWELL.

THE use of an optical pyrometer to determine the temperature of a metal gives an approximation to the true temperature only when the metal is enclosed, as in a furnace, the walls of which are practically all at the same temperature. The more completely the enclosed region above the metal surface approximates a uniform temperature, the more accurate will be the temperature given by the pryometer. If the metal is in an open crucible with its surface radiating freely into open space and receiving no light from the surroundings there is then a complete departure from black-body conditions. The radiation is selective in such a case and is characteristic of the particular surface. An optical pyrometer sighted upon such a surface will always give temperatures too low. The corrections which must be made to such optical readings have been determined in only a few cases. And the comparisons which have been made by various observers are in rather poor agreement. Recent observations by Stubbs and Prideaux[1] on the emissivity of gold near its melting point and by Stubbs[2] on silver and copper show great changes in emissivity with wave-length. This may account in large measure for the lack of agreement of different observers. The absorbing glasses used in the various pyrometers are never strictly monochromatic. One red glass, admitting a wider range of light than another, may give quite a different value for the black-body temperature. In order to extend our knowledge concerning the laws of selective radiation from metallic and other surfaces there is need of more observations made with monochromatic light. The data submitted in this paper are not intended to meet that requirement fully but they may be of some practical value.

The author has made comparisons of actual and black-body temperatures for several metals through a wide temperature range. A Morse pyrometer, which is of the disappearing filament type, was used with a red absorbing glass. The glass used is one of the best obtainable as regards mono-chromatism. The per cents. of light of the various

[1] Proc. Roy. Soc., Ser. A, Vol. 87, Oct. 1912, p. 451.
[2] Proc. Roy. Soc., Ser. A, Vol. 88, Mar., 1913, p. 195.

wave-lengths transmitted through this glass are given in Fig. 1. The corrections to optical readings which the author has determined for various metals are approximate for any pyrometer in which an absorbing glass transmitting between 0.60μ and 0.70μ is used. They are probably exact only when the absorbing glass is exactly like the one here used. The comparisons are shown for silver, gold, copper, nickel and iron in Fig. 2. Using data from the curves in Fig. 2 the author has computed relative emissivities for these metals through a temperature range of more than 1000 degrees. The relation between relative emissivity and temperature is shown in Fig. 3.

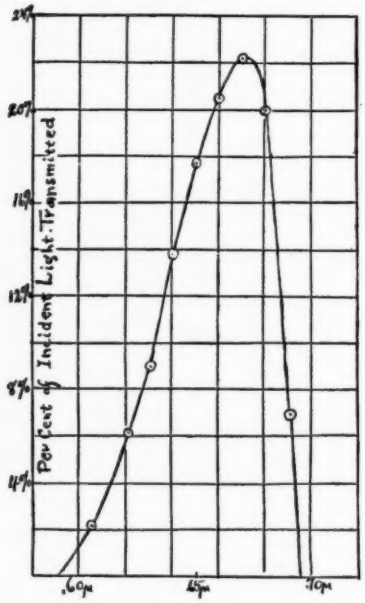

Fig. 1.

Transmission through the absorbing glass used in the Morse pyrometer.

The methods by which actual temperatures were obtained for the various metals in the solid and liquid states, the precautions required in individual cases and the method of computing emissivities are discussed in the following pages.

The pyrometer was calibrated to read "black-body" temperatures first, by sighting into a Reichsanstalt "black-body," the temperature of which was determined by means of a platinum, platinum-rhodium thermo-junction which had been calibrated by the Bureau of Standards. Calibration points as high as 1600° C. were obtained in this way. Beyond this a calibration point was obtained at the melting point of platinum which was taken as 1775° C. A piece of pure platinum was placed in a carbon cavity with a small opening at the top. A small platinum wire projected through this opening from the top of the specimen within. Melting of the specimen was shown by movement of the wire. The cavity was heated electrically and the pyrometer sighted into the opening from above. Additional points between 1600° C. and 1800° were obtained by use of a Wanner pyrometer, for the calibration of which only two points are necessary, the extrapolation being based on Wien's law. The Wanner and the Morse were sighted simultaneously into the black-body cavity in the carbon rod. Calibration points for the Morse were thus obtained at 1625° C., 1725° C. and 1810° C. The point obtained on the melting-point of platinum was in very good agreement with these

points. The author's calibration was found to agree exactly with that furnished by the manufacturer.

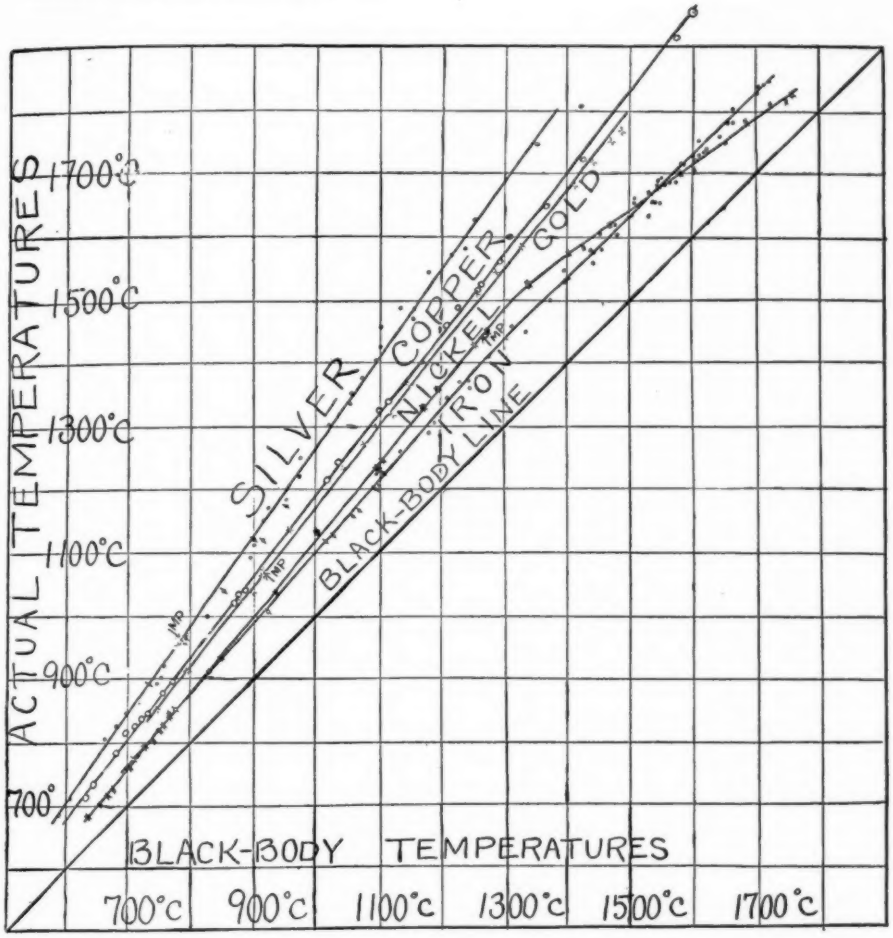

Fig. 2.

Black-body temperatures as compared with actual temperatures for various metals. The deviations of optical readings from actual temperatures are indicated by the vertical heights above the 45 degree, or "black-body" line.

Silver.—Points indicated by small circles with lines were taken with a platinum, platinum rhodium junction encased in quartz. The other points indicated by small circles were taken as described in the text.

Copper.—Points indicated by large circles.

Gold.—Points indicated by crosses.

Nickel.—Points indicated by double circles with crosses were taken by means of a carbon-graphite thermo-junction, the tips of which were frozen into the metal as it solidified from the molten state. The other points for nickel are indicated by small circles with crosses. These were taken as described in the text.

Iron.—Points indicated by small circles were taken on molten iron, those indicated by small circles with lines were taken on solid iron.

The real problem involved in the work was the determination of actual temperatures. The material to be studied was placed in a small cavity in a carbon rod which was heated electrically by a current from a low tension transformer. Observations were made on the free surface of the heated substance and actual temperatures taken simultaneously by means of thermo-junctions or otherwise as described later. Care was

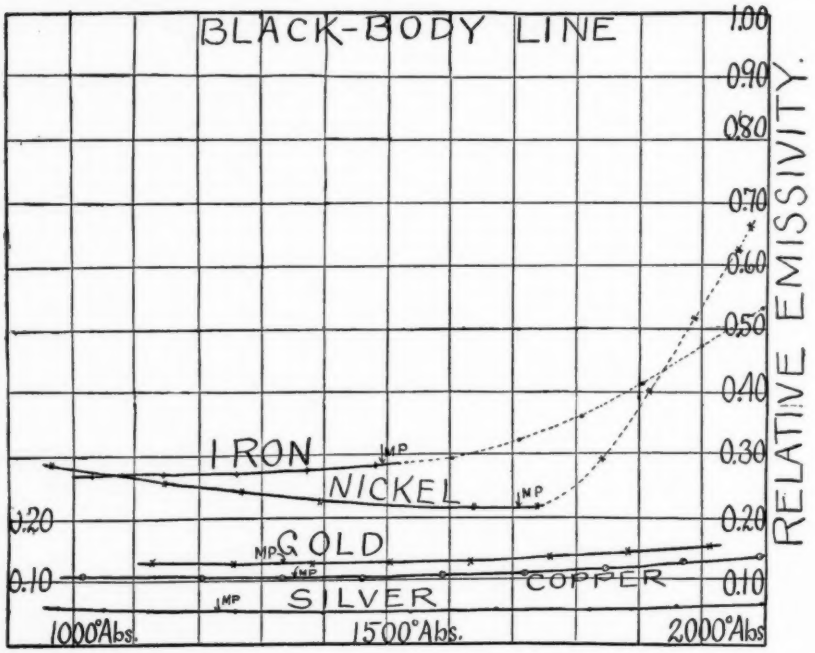

Fig. 3.

Relative emissivity and absolute temperature. (Light of wave-length approximately 0.66μ.)

taken that the material not only filled the cavity but projected above the upper surface of the carbon rod, thus avoiding any chance of light reaching the surface from the surroundings. There were no surrounding walls, the carbon being heated in the open air. Thus the radiation from the surface was that of a body radiating freely into open space and receiving and reflecting no light from its surroundings. The transformer was of 40 kilowatt capacity and used on a 2200-volt circuit. It was capable of stepping the voltage down to 30 in the secondary when the latter was on open circuit. With the secondary short-circuited through the carbon heating rods probably several hundred amperes were ordinarily required. By using carbons of different sizes any desired temperature between 600° C. and 2200° C. could be obtained.

For the determination of actual temperatures a thermo-junction of platinum and platinum with 10 per cent. rhodium, fused into the surface, offered a satisfactory method for solids, but, for convenience and rapidity, optical readings in a hole about 2 mm. deep by 1 mm. wide were taken as giving true temperatures for the surface. The hole was bored with a drill which made the bottom conical. A test of these readings was made by fusing a thermo-junction into the edge of the cavity about flush with the surface. The observations taken in this test are as follows:

Optical Readings in Cavity.	Thermojunction Readings.
1003° C.	990° C.
1005	1000
1015	1010
1050	1045
1075	1060
1130	1125

These readings are seen to agree within the range of accuracy possible with an optical pyrometer. The advantage of the optical method lay in the fact that the pyrometer could be sighted, first on the cavity, then on the surface, then back on the cavity, all within a few seconds and with the one instrument, while thermo-junction measurements involved balancing a potentiometer, which required considerably longer time. And absolutely steady conditions for any length of time were quite out of the question.

In the observations both on solid and molten metals extreme precautions were taken to obtain clear uncontaminated metallic surfaces. In the case of molten metals little trouble was experienced since slag and oxide always float to the edges of the crucible leaving a clear brilliant surface in the middle of the globule. A study of the surface of solid metals brought out the fact that the radiation was altered very greatly by slight changes in the surface. The brilliancy of the polish, the presence of very slight amounts of the polishing materials (rouge, etc.), or an imperceptible film of oxide changed the radiation decidedly. Consistent, reproducible results were only obtained on surfaces procured by solidification from the molten state, the freezing taking place in an atmosphere of hydrogen or nitrogen and the metal kept in such an atmosphere throughout the observations.

With such surfaces no abrupt change in the character of the radiation on passing from solid to liquid or vice versa was observed. A molten metal on freezing will ordinarily be observed to flash up brightly. This does not occur in a reducing atmosphere unless undercooling has taken place.

MOLTEN IRON.

In the case of molten iron a small carbon cavity was immersed in the metal with the opening in the cavity flush with the surface of the iron. Thermo-junction tests showed that cavity readings taken with the pyrometer indicated true temperatures at the surface very accurately. For these observations a large graphite crucible (about 5 cm. deep and 5 cm. inside diameter) was placed in a trough between fire-clay bricks and the trough filled with granular carbon packed tightly around the crucible. Current from the low tension transformer passed through this tightly packed carbon would quickly bring it to any desired temperature. A small carbon rod was fitted into a hole in the bottom of the crucible in such a way as to project vertically up through the middle of the crucible until flush with the top. A small cavity 5 mm. deep by 2 mm. wide drilled into the top of this rod constituted the black-body. The crucible surrounding the rod contained the molten iron. Simultaneous readings were taken at various temperatures in cavity and on iron surface.

As a check on the true temperature, a platinum, platinum-rhodium junction encased in a quartz capillary was pressed into the surface and the temperature readings thus taken compared with the optical readings on the free surface. Extended observations were not possible owing to the softening of the quartz and consequent destruction of the junctions. Two readings which were taken under steady temperature conditions are as follows:

Surface reading....1250° C.	Thermojunction reading....1360° C.
Surface reading....1275° C.	Thermojunction reading....1385° C.

This shows surface readings to be about 110° low. The cavity indicates surface readings to be about 130° low in this temperature region. It is to be expected that the junction readings would not be high enough. The junction was simply laid on the surface and pressed down but not immersed and there must be a temperature gradient from the iron surface through the quartz to the junction. If the junction is pressed beneath the surface, readings depend entirely on the depth of immersion and are no indication of the surface temperature. Hence the cavity readings being slightly higher than the junction readings, as taken above, are to be regarded as probably the truest indication obtainable of the surface temperature. Considerable difficulty was experienced in keeping conditions steady long enough to get satisfactory readings. The best results were obtained while the temperature was slowly rising or slowly falling. The cavity reading was first taken then the surface, and finally the cavity again. The mean of the cavity readings was taken as giving the actual temperature of the surface. The following set of data is a sample of

many runs taken on molten iron. The curve for iron (Fig. 2) records the data of six such runs covering the temperature range from 1200° C. to 1850° C.

Sample Set of Readings on Molten Iron.

Actual Temperatures.		Black Body Temperatures of Surface.
Cavity Readings.	Mean.	
Temp. rising:		
1550°–1580° C.	1565° C.	1440° C.
1580 –1625	1603	1480
1625 –1645	1635	1500
1645 –1665	1655	1505
1665 –1655	1660	1505
1655 –1655	1655	1535
1655 –1685	1670	1530
1685 –1705	1695	1552
Temp. falling:		
1425 –1395	1410	1272
1395 –1320	1357	1240
1320 –1255	1287	1175
1225 –1185	1205	1095

Solid Iron and Other Metals.

For solid iron the surface studied was obtained by solidification from the molten metal. A small carbon rod was punched into the metal as it was solidifying and in the pasty condition. The frozen surface layer was thus pressed down and the still liquid mass below caused to flow up over it from around the edges and while so doing to solidify. All was carried on beneath burning hydrogen. A brilliant reflecting surface was thus obtained. Readings on such a surface give a curve continuous with the curve for the liquid surface (see Fig. 2). A surface obtained by simply high burnishing does not retain its luster and does not give a curve continuous with that for the molten metal. The cavity used for the "black body" was the hole left by the carbon rod mentioned above. The same method was used for all the metals studied. Only thus could a clear uncontaminated metallic surface be obtained. In some cases the metal was cooled through change of state and down to room temperature in a hydrogen or nitrogen atmosphere and a small hole then drilled into it to serve as a black-body. Only surfaces carefully protected from oxidation give consistent reproducible results.

Molten Metals—Gold, Silver, Copper and Nickel.

For gold, silver, nickel and copper in the molten state actual temperatures were determined by means of a thermo-junction of carbon and graphite.

A suitable junction of these materials as finally developed in the course of this work was composed of rods of carbon and graphite about 30 cm. long by 2 mm. diameter clamped side by side but insulated from each other by an air space. The junction was closed across the surface of the metal into which the rods were plunged. The cold ends of the rods were fitted into heavier pieces of carbon and graphite at the farther ends of which were fastened the copper wires which completed the circuit through the galvanometer or potentiometer. For a further description of these junctions with methods of calibration and precautions to be taken in their preparation and use, see following article on "Note on Carbon-Graphite Thermo-junction."

A small globule of the silver or other metal was placed in a cavity bored in the side of a carbon rod, the carbon rod being fitted horizontally into graphite electrodes and heated electrically. The cavity was usually about 10 mm. wide by 6 mm. deep. Enough metal was used to fill the cavity completely with the rounded surface protruding well above the surface of the rod. The thermo-junction points were adjusted to just penetrate the surface. Optical readings were made on the surface as close as possible to the junction. The thermo-electric circuit was completed through a resistance of 10,000 ohms with a high resistance galvanometer shunted around 1000 of these. The galvanometer deflections were calibrated to read temperatures direct. Several runs were made in this way on each metal from temperatures ranging from the melting point to 1900° C. As the junction rods oxidized rapidly in the air above the metal new junctions were required for each run. Hence the data is not dependent upon a single run and a single calibration but in each case on several different junctions each independently calibrated.

On molten silver a set of observations was taken in which actual temperatures were measured with a platinum, platinum-rhodium thermo-junction encased in a very fine quartz tube, the junction being pressed upon the surface of the metal. These readings are, as was expected, slightly lower than the carbon junction measurements (see Fig. 2). On solid nickel a set of observations were taken, in which actual temperatures were measured by means of a carbon-graphite junction which was "frozen" into the metal as it solidified from the molten state. These readings (see Fig. 2) agree well with the observations taken by the "black-body" cavity method.

RELATIVE EMISSIVITY.

The relative emissivities for the different metallic surfaces for the whole temperature range from 600° C. to 1850° C. were computed in the follow-

ing way. If I_a is the intensity of light emitted by any hot body and T_a the corresponding "black-body" temperature, by Wien's law the relation between the two is

$$I_a = c_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T_a}} \tag{1}$$

For a full radiator at the same actual temperature we have

$$I = c_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T}} \tag{2}$$

Dividing (1) by (2) we get

$$\frac{I_a}{I} = \frac{e^{\frac{C_2}{\lambda T}}}{e^{\frac{C_2}{\lambda T_a}}}. \tag{3}$$

Now $I_a/I = E$ (relative emissivity). We may then write (3) as

$$\log_{10} E = \frac{C_2 \times .434}{\lambda} \left(\frac{T_a - T}{T T_a} \right). \tag{4}$$

If C_2 is taken as 14,500 and λ as 0.66μ then (4) becomes

$$\log_{10} E = 9{,}535 \left(\frac{T_a - T}{T T_a} \right).$$

Thus (E) may be determined.

This expression requires the light to be monochromatic and the question now arises whether the light admitted to the pyrometer field is sufficiently monochromatic to justify its use. Fig. 1 shows that about 80 per cent. of the light is between wave-lengths 0.635μ and 0.685μ, with the average at 0.660μ. Computations of the emissivity of nickel at 2078° absolute (using data from the curve, Fig. 2) on the basis that the light is all of wave-length 0.635μ give 0.653 for E. On the basis that the light is all of wave-length 0.685μ we get for E the value 0.673. As the light is fairly evenly distributed between these limits the value of E for the whole band cannot be far from the mean of these values, which is obtained if the light is taken as of wave-length 0.66μ.

A more extreme case is that of silver at 1823° absolute. If all light were of wave-length 0.635μ (E) would be 0.051; if it were all of wave-length 0.685μ, (E) would be 0.063. The value of (E) must lie between these limits and the mean is obtained if we assume the light to be all of wave-length 0.66μ. The error introduced by regarding the light as monochromatic and of wave-length 0.66μ is therefore small and the use of Wien's equation is justified.

Fig. 3 shows the relation between relative emissivities and absolute temperatures. The low emissivity of silver is remarkable. At 1800° C. it radiates like a "black-body" at 1400°. Its emissivity is only about

5 per cent. that of a full radiator. Through the red glass the molten silver appears as a dark spot surrounded by the white hot carbon, yet carbon and silver are at approximately the same temperature.

Molten nickel takes up carbon from the crucible rapidly and beyond 1800° C. becomes a semi-solid pasty mass. On cooling the carbon which was either in solution or combined as carbide crystallizes out as graphite the mass becoming fluid again. The freezing point appears not to be affected by this absorption and expulsion of carbon. It is probable that the emissivity of molten nickel is modified by this absorbed carbon, the sharp rise shown on the curve being due to an actual change in composition which might not occur if the nickel were melted in some other sort of container. The effect was observed with iron also but not at all with gold, silver or copper. On account of this effect the emissivity curves for molten iron and nickel are given as dotted lines since they probably represent changes due to change in composition of the metal.

The relative emissivity which by Kirchhoff's law is the same as the absorptivity is shown to be practically constant for silver, copper and gold through the whole temperature range from 600° C to 1800° C. The values agree also with absorptivities at room temperatures as determined by Hagen and Rubens.[1] The table shows a comparison of Hagen and Rubens' values at 25° C. and the author's values at high temperatures. For silver, copper and gold a knowledge of the reflecting power at ordinary temperature is all that is required for a determination of optical

	Hagen and Rubens — Absorptivity.			The Author.
	0.60 μ.	0.65 μ.	0.70 μ.	0.66 μ (Approx.)
Silver	.074	.065	.054	.055
Gold	.156	.111	.077	.125
Copper	.165	.110	.093	.105
Nickel	.35	.34	.31	[.355–(?)]
Steel	.45	.44	.42	.27–.29 (Iron)

pyrometer corrections at any temperature. The author believes that this has never been verified through a wide temperature range before. Unfortunately for the generality of the above relation, nickel seems to have a temperature coefficient of relative emissivity. Taking the value of this coefficient from the curve as $-$ .000125, the value of the relative emissivity for 25° C. is found by computation to be .355 per cent. The fact that Hagen and Rubens have found practically that value is rather interesting.

[1] Annalen der Physik, 8, 912, p. 1.

All the curves show some evidence of an increase in emissivity at extreme high temperatures.

With regard to emissivity and change of state there is no evidence here of a sudden change occurring, as has been reported by some observers. Attention is called to the data taken on both the molten and solid metals near the melting points, especially for silver, gold and nickel. Except for the movement of slag at the edges of the liquid globule it is impossible to tell optically whether the metal is liquid or solid when near the melting-point. Surfaces artificially prepared, on the other hand, no matter how brilliantly polished always show a sudden drop in emissivity on melting. When a molten metal solidifies *in air* a rise in emissivity occurs as shown by a sudden brightening or flashing up. Evidently these changes are due to the formation of an oxide film or, in the case of the prepared surfaces to the presence of foreign material in the "pores" of the metal.

CORNELL UNIVERSITY,
February, 1914.

NOTE ON A THERMO-JUNCTION OF CARBON AND GRAPHITE.

BY CHARLES C. BIDWELL.

IN the course of certain studies on the radiation from molten metals[1] the need of some means of measuring temperature where optical methods were not applicable led to the utilization of the thermal E.M.F. between carbon and graphite and the construction of thermo-junctions, by means of which temperatures as high as 2200° C. were read. Although there is no mention in the scientific literature of the use of such a junction, it has been learned since the above mentioned work was performed that the Bristol Mfg. Co., of Waterbury, Conn., holds a patent granted in 1905 on a thermo-junction of carbon and graphite with adjustable tips. They have never developed the junction into a commercial instrument, probably on account of the variation in E.M.F. which necessitates constant re-calibration and is sufficiently serious to preclude its use in industrial processes. The junction however offers certain possibilities in the region beyond the reach of the platinum, platinum-rhodium couple in cases where radiation methods cannot be used. For scientific purposes therefore it may be valuable and worth a description.

The junction, as finally constructed, was designed to measure the temperature of molten metals. It was composed of rods of carbon and graphite about 30 cm. long by 2 mm. diameter clamped side by side but insulated from each other by an air space. The ends that dipped into the metal were narrowed to points and nearly met where they penetrated the surface of the metal. The thermo-electric circuit was thus closed through the metal. The cold ends of the rods were fitted into heavier pieces of carbon and graphite, thus giving each an additional length of 50 cm. At the farther ends of these heavier rods, were fastened the copper wires which completed the circuit through the galvanometer or potentiometer. The copper-carbon and copper-graphite contacts which constituted the cold junctions were wrapped in asbestos and maintained at room temperature.

Certain precautions were required in the preparation and use of these junctions. The E.M.F. is large, approximately 0.02 microvolts per

[1] See preceding article on A Comparison of Actual and Black-Body Temperatures.

degree and very nearly linear up to 2000° C., but rather erratic and untrustworthy if the carbons and graphites are used without special previous heat treatment. Electrically heating to about 2000° C. for several minutes seems to drive off impurities. At least the E.M.F. thereafter is fairly steady. If such a junction is now calibrated it will be found that the calibration will hold as long as the tips last. The rods oxidize in the air just above the crucible, the tips beneath the metal undergoing little change. If the tips are broken off and the junction rods pushed down into the metal again the original calibration may or may not be correct; usually a new calibration must be made. As long as the tips last the calibration is good.

The variation of the thermal E.M.F. with different rods and for different regions along the same rod may be due to partial graphitization

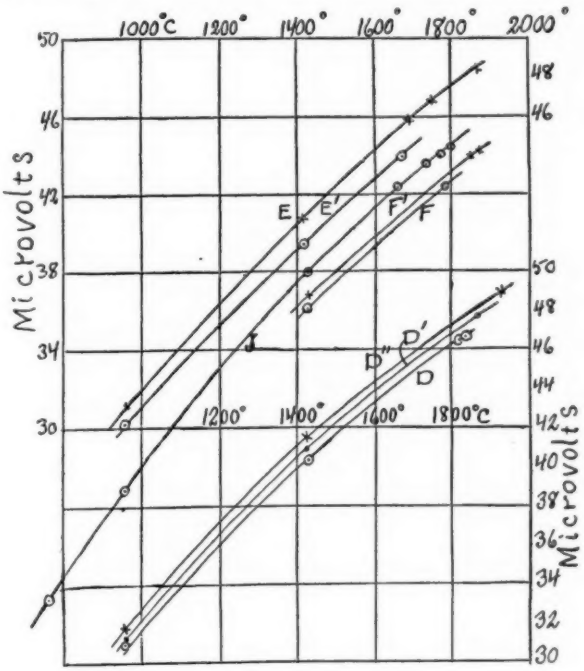

Fig. 1.
Calibration curves for carbon-graphite thermojunctions.

of the carbon or to incomplete graphitization of the graphite. Both the carbon and graphite seem to be at fault. The carbon rods were ground down from National Carbon Co. electric light carbons, the graphite was the artificial product made by the Acheson Graphite Co. The extent of the variation may be seen by consulting the calibration curves for the rods actually used in the author's work on optical and black-body temperatures of metals. These are shown in the figure.

The curves marked D, D', D'' are calibrations for the same pieces of graphite and carbon. D was for the original pieces; D', the calibration after the original tips were oxidized away and the rods again ground down to points; D'' after regrinding a second time. Likewise E and E' are calibrations for another junction, E' after grinding down new tips. Similarly, F and F'. It is evident that the variation in E.M.F. even for a few millimeters along the same piece of carbon or graphite is large enough to invalidate temperature readings unless the calibration is constantly checked.

For calibration the junction was immersed in molten silver which was allowed to cool through its freezing point. A high resistance galvanometer in series with the junction was observed during the cooling. At the freezing point the motion was arrested, the reading remaining stationary for several seconds. The point for 960° C. was thus obtained. The process was repeated with nickel. Undercooling occurred usually with nickel and was so considerable that the galvanometer could not keep pace with the quick rise in temperature which occurred when the solidification did take place, and the cooling would begin again before the galvanometer had swung back to the melting-point. Cooling curves were taken to determine the seriousness of this lag. It was found that the galvanometer would usually get back to within 10 degrees of the melting-point when cooling began again. 1430° C. was taken as the melting point of nickel and allowance made for the lag when undercooling occurred. For higher temperature points the junction tips were immersed in a globule of molten iron, optical readings taken on the surface, and the correction as already determined for iron applied. Calibration points in iron as high as 1900° C. were thus obtained.

If samples of carbon and graphite of greater purity than was available for this work could be obtained and the filaments protected where exposed to oxidation a serviceable instrument might be made of this junction.

CORNELL UNIVERSITY,
February, 1914.

DISCHARGE POTENTIALS ACROSS VERY SHORT DISTANCES.

BY EDNA CARTER.

PASCHEN'S law states that the potential required to produce a discharge through a gas is a function of the product ($p.d$) of the pressure and the distance between the electrodes. Paschen[1] worked with pressures above the critical pressure and found the potential very nearly proportional to $p.d$. Carr[2] extended the measurements to lower pressures and smaller distances and found that the potential reached a minimum value and then increased with a further decrease in $p.d$ and finally it was very nearly inversely proportional to this product. The work of Almy[3] and Williams[4] indicates that this would also be true even at atmospheric pressure, if the distances were made sufficiently small and the discharge could be forced to go across the shortest distance.

By means of a Chamberlain[5] compound interferometer with its massive carriage and accurate ways, it was quite easy to test these results and show that down to a distance of 0.5 wave-length of Na light the minimum spark potential of about 270 volts is still required to produce a discharge in dry air. Williams has proved that in this case the spark does not pass across the shortest distance. It was hoped that measurements could be made inside of 0.5 wave-length with this interferometer which can be made to measure distances to 1/40 of a wave-length, but it was impossible to say that the sparks which were obtained over shorter distances at lower potentials were not due to the yielding of the material under the electrostatic force so that the electrodes were first brought into contact.

Meurer[6] was unable to detect any effect of a magnetic field, transverse or longitudinal, on the potential required to start a spark discharge in air at atmospheric pressure, although such effects are obtainable at reduced pressures. It was thought that at the knee of the curve an effect might be found upon the minimum spark potential. To investigate

[1] Wied. Ann., 37, p. 79, 1889.
[2] Phil. Trans., *A*, Vol. 201, 1903.
[3] Phil. Mag. (6), Vol. 16, 1908.
[4] PHYS. REV., 31, p. 216, 1911.
[5] PHYS. REV., 23, p. 187, 1906.
[6] Ann. der Physik, 28, p. 199, 1909.

this point an electromagnet was made out of a rod of iron 1.5 cm. in diameter cast into the form of a ring about 15 cm. in diameter and wound with two layers of heavy copper wire. A gap 1.5 mm. wide was left in the ring and the iron at the gap was tapered to half its diameter. The spark gap between the iridium electrodes was introduced into this space so the magnetic field was transverse. No attempt was made to measure the strength of the magnetic field, but a current of 30 amperes through the electromagnet produced no measurable effect upon the discharge potential.

J. J. Thomson in his "Conduction of Electricity through Gases" has shown how on theoretical grounds one might expect the curve showing the relation between potential and $p.d$ to have two branches. Assuming that the curve obtained in Carr's experiments has become the hyperbola demanded by the Thomson theory for small values of $p.d$ and extrapolating for still smaller values, Madelung[7] found that the potentials obtained by experiment were very much lower than those demanded by the theory. When the pressure was reduced to about 0.07 mm. with a distance between the electrodes of 0.05 mm., the discharge took the form of a spark across the shortest distance. For this value of $p.d$ the theory requires a potential difference of about 60,000 volts. In reality it was only 9,000 volts. As the pressure was reduced in Madelung's experiments, the potential did not increase proportionally but seemed to reach a limiting value which was very nearly proportional to the spark length. This limiting value of the potential gradient he sets at 400,000 volts pro cm. He extends his curves showing the relation between potential and spark length to the origin.

Perhaps we ought not to expect Paschen's law to hold for these low pressures on account of the probable lack of uniformity in the distribution of the gas. It seemed of interest however to know what happens at distances equal to a few wave-lengths of Na light and also to find out if there is a limiting value of the potential gradient when every effort is made to rid the electrodes of gas. It was also hoped that some light would be thrown on the questions concerning electrical contact and the expulsion of electrons by metals.

An arrangement was attached to a Chamberlain interferometer in such a way that by means of a separate screw and a flexible copper disk two iridium electrodes, about 1 mm. in diameter, the one pointed and the other flat, could be brought together and separated inside the vacuum. With the aid of an additional iridium contact outside the tube, the distance apart of the electrodes could be measured in wave-lengths with as much

[7] Phys. Zeits., Feb., 1907.

accuracy as has been warranted so far by the results. The electrodes were contained in a glass tube about 2 cm. long and 2 cm. in diameter. The discharge could be observed in a low power microscope. The necessary voltages were obtained from a 1,600-volt battery of small accumulators and a high potential machine capable of furnishing any voltage up to 4,000. A Braun electrometer was used to measure the potential difference. A graphite resistance of 10^7 ohms was contained in the circuit and there was a cadmium iodide resistance permanently connected to the machine. The vacuum was secured by means of a Gaede mercury pump and cocoanut charcoal cooled in liquid air. One cannot claim for this anything higher than 0.0001 mm. The state of the vacuum was judged by the discharge through an attached Pflücker tube with large capillary. With the best vacuum conditions no visible discharge was produced in this tube by an induction coil capable of furnishing 60,000 volts.

When the tube was first evacuated, bluish white sparks passed rather easily and very irregularly. They resembled the explosions of occluded gases which one sees on the electrodes in any discharge tube. As the vacuum was maintained a longer time and the discharge passed through at intervals, these irregular sparks became less frequent. The discharge almost invariably began with a faint glow on one or both electrodes, which grew brighter as the electrodes were made to approach each other; and often after long continued maintenance of the vacuum the two glows simply melted together without any sudden intense lighting up of the field and disappeared at contact. When the graphite resistance was removed, the glow appeared as usual and just before contact very brilliant sparks of the same color were seen. These were surely due to the metal vapor. On one or two occasions yellowish sparks distinctly different from the white ones were observed. These may have been due to impurities in the metal or to the presence of some other gas. The phenomena were very irregular, but the distance at which the glow first appeared and a galvanometer gave the first indication of a current was fairly well defined and was approximately proportional to the voltage used.

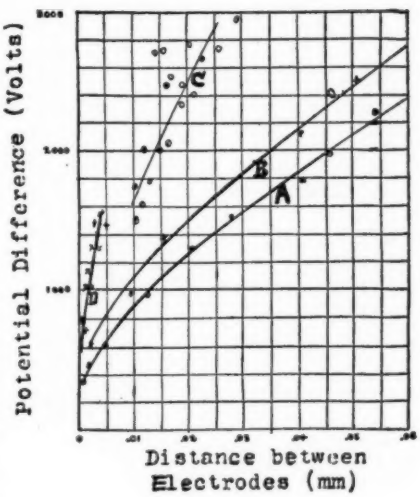

Fig. 1.

The curves represent the relation of the voltage to the distance between the electrodes when the glow first appeared. Curve *A* was obtained soon after the tube was evacuated and curve *B* a day later. The values in the neighborhood of *C* were obtained with the high potential machine after various means had been used to remove the gas from the electrodes. The values about the curve *D* were obtained with the battery after using what seemed to be the most effective method of removing the gas. Enough air was introduced so that a glow discharge was produced across the gap by an induction coil. After allowing this discharge to pass for several minutes the tube was quickly evacuated again. Taking out the graphite resistance and allowing a series of brilliant sparks to pass across, while the high vacuum was maintained, seemed to increase also for a time the potential gradient necessary to start the discharge. Later the electrodes were inserted in a quartz tube about 20 cm. long and 0.5 cm. in diameter where by means of an electric furnace they could be heated for hours to a bright red heat with the evacuation going on at the same time. The heating did not seem to have so much effect as the earlier methods and the phenomena in this smaller tube were more irregular, the discharge having a tendency to start at points back on the copper wires to which the iridium electrodes were soldered. Keeping away the mercury vapor by inserting a tube constantly surrounded by liquid CO_2 in acetone produced no noticeable effect.

The results show a minimum discharge potential in the neighborhood of 300 volts. Curve *A* corresponds rather closely to the potential gradient set by Madelung as the limiting value. Curve *C* gives a potential of 10^6 volts pro cm. and curve *D* 3.10^6 volts pro cm. Since the results are so irregular and seem to depend upon the condition of the electrodes rather than the exact state of the surrounding vacuum and since the value of the potential gradient seems to be increased by any means which would drive gas out of the electrodes, it is very probable that the discharge takes place in a gas of considerably greater density than is indicated by the state of the vacuum. This gas may permeate the electrodes but it also seems to cling fast to their surfaces. The presence of this gas may make impossible a complete confirmation of the Thomson interpretation of Paschen's law for small values of $p.d$, but another approximation may be made by using tungsten electrodes which can be more readily and more nearly completely freed from gas.

These investigations were carried out in the physical laboratory of the University at Wurzburg. My sincerest thanks are due Professor Wien and the other members of the department for their stimulating interest and helpful advice.

VASSAR COLLEGE.

NOTE ON THE FLUORESCENCE OF FROZEN SOLUTIONS OF THE URANYL SALTS.

By Edward L. Nichols and Ernest Merritt.

THAT aqueous solutions of the uranyl salts exhibit fluorescence similar to that of the salts themselves but of a feebler intensity has long been known. To determine whether the nearly equidistant bands, of which in general the fluorescence spectra of these solutions consist, retain their character when the solutions are frozen and reduced to the temperature of liquid air or whether resolution into groups of narrower bands occurs, was the purpose of the experiments to be described in this preliminary note.

To this end solutions of uranyl sulphate, uranyl potassium sulphate, uranyl nitrate and uranyl acetate of widely varying concentrations were used.

The solutions were placed in a test-tube *F* (Fig. 1) within a cylindrical

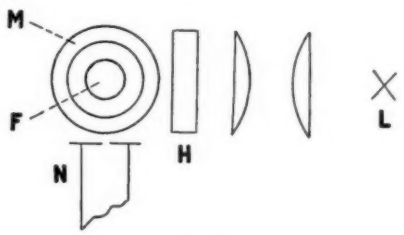

Fig. 1.

Dewar flask *M* with unsilvered walls. Fluorescence was excited by a conical beam from a right-angled carbon arc *L* and the spectrum was observed with a Hilger spectroscope, the collimator of which is shown at *N*. For this spectroscope a suitable spectrograph could be substituted when photographs were desired.

A ray filter *H* absorbed all light from about wave-length .47 μ to the extreme red so that the fluorescence bands appeared upon a dark background.

In no case, with the exception of uranyl acetate in alcohol, was a resolution of the fluorescence of aqueous solutions into narrow line-like bands observable; but there were certain striking changes in the character of

the spectra when temperature and concentration were varied. These are noted briefly in the following paragraphs:

A. AQUEOUS SOLUTIONS.

1. *Uranyl Sulphate.*—The solutions of this salt present the simplest case; that of a single set of rather broad bands essentially unmodified either by temperature or dilution excepting as to breadth and intensity. In this preliminary survey a set of solutions were made by diluting the concentrated solution successively with ten parts of water. Measurements were made of the stronger bands at the temperature of liquid air to determine approximately the location of the crests.

The results are given in Table I. In this table the bands measured are designated arbitrarily by letters *c*, *d*, *e*, *f*, and *g;* there being two bands, *a* and *b*, of longer wave-length discernible in the spectroscope which do not appear in the photographs.

TABLE I.

Uranyl Sulphate in Water —185° C.

Concentration.	Band *c*.	Band *d*.	Band *e*.	Band *f*.	Band *g*.
1.00	.5631μ	.5380μ	.5131μ	.4915μ	.4807μ
0.10	.5625	.5377	.5130	.4915	.4797
0.01	.5626	.5380	.5131	.4916	.4796
0.001	.5626	.5376	.5129	.4915	.4804
0.0001	.5632	.5388	.5138	.4929	?

Estimations of width were also made but these are not included in the table because in a photograph the apparent width of a band depends on the intensity of the exciting light, the time of exposure and the development of the negative. As development progresses the crests appear first and darkening extends laterally outwards in both directions.

The widths as they appear in the photographs were roughly about 60 Ångström units for band *c*, 80 units for bands *d*, *e* and *f* and 50 units for band *g*.

Within the errors of observation, which would appear to be one or two Ångström units in the case of the more intense bands *e* and *f*, two or three units for band *d*, three or four units for band *c* and perhaps ten units for band *g* which was very vague, broad and weak, there is no shift with the concentration, ascertainable by this method, until a dilution to .001 is passed. In the weakest solution (.0001) however the bands are shifted to the red. The movement is about fourteen units for band *f*, eight or ten units for bands *d* and *e* and not more than five units for band *c*. The location of band *g* for this solution was not possible on

account of weakness of the spectrum and the disturbance due to the presence of the neighboring absorption band of liquid air.

No attempt was made to secure photographs of the spectra of still weaker solutions, but a series of visual observations made by freezing and exciting aqueous solutions each of which contained twice the proportion of water of the preceding one showed that the two brightest bands (*e* and *f*) were still dimly visible when a solution containing *one part of the concentrated solution to 500,000 of water* was excited at the temperature of liquid air.

2. *Uranyl Nitrate.*—The concentrated aqueous solution of this salt when frozen and excited to fluorescence showed a double series of bright bands alternately narrow and broad. The narrower bands were on the side towards the violet, perhaps a third as wide as the accompanying broad band and less intense. The spectrum increased rapidly in brightness as cooling proceeded.

When the solution was diluted to 1/10, the fluorescence on cooling first appeared as a single very broad continuous band from red to blue but was rapidly resolved into a double series as in the spectrum of the concentrated solution.

With further diminution of concentration the double banded spectrum is observed although not so bright; at least until 1/1000 is reached. In solution with 10,000 parts of water however the narrower bands could no longer be distinguished.

The location of the bands, in the case of the nitrate, as shown in Table II. is somewhat less satisfactory than with the sulphate. The drift towards longer wave-lengths as the dilution increases is however not less marked. Band *g*, as in Table I., is encroached upon by the absorption

TABLE II.

Uranyl Nitrate in Water at —185° *C.*

Conc.	Band.		Band.		Band.		Band.		Band.	
	c.	*c'.*	*d.*	*d'.*	*e.*	*e'.*	*f.*	*f'.*	.	*g'.*
1.00	.5681μ	.5596μ	.5430μ	.5334μ	.5185μ	.5092μ	.4964μ	.4873μ	.4802μ	.4724μ
0.10	.5682	.5595	.5435	.5338	.5190	.5091	.4970	.4870	.4799	
0.01			.5432	.5330	.5196	.5089	.4970	.4875	.4804	
0.002					.5207	.5103	.4977	.4882	.4803	
0.001					.5211	.5100	.4985		.4805	

band of liquid air to such an extent that the location of the crest is open to a considerable correction and the apparent absence of drift is probably not significant.

3. *Uranyl Acetate.*—The behavior of frozen solutions of uranyl acetate is much more interesting and remarkable than that of the sulphate or nitrate. In concentrated solution this salt has a fluorescence spectrum which to the eye appears to consist of a single very broad and faint band. At the temperature of liquid air this is resolved into a group of brilliant bands nearly equally spaced and in their general appearance similar to those of the sulphate but differently located (see Table III.). Upon dilution to 1/160 the resolved spectrum consists of broad very strong bands between which, at very low temperatures, much narrower and very much fainter bands appear.

Further dilution to 1/1600 produces another marked change. The broad and narrow bands are now supplanted, as if by partial fusion, by very broad and rather feeble bands having an intermediate position. Of the two which it is possible to locate with certainty in the photograph taken of the spectrum of this solution the one of shorter wave-length is clearly double although overlapping, having broad crests at approximately .5086 μ and .5215 μ. The other broad band having its middle about .5491 μ cannot be even partially resolved. The weakest solution upon which observations were made had a concentration of 1/16000. Its spectrum consisted of broad dim unresolved bands, the two strongest of which had their crests at about .5024 μ and .5239 μ. The outlying bands in the neighborhood of .48 μ and .54 μ were too vague to be located in the photograph.

The bands upon which measurements were attempted are given in the following table. Whether the changes in position should be ascribed, as in the case of the other salts, to a shift with concentration can be determined only by further observations.

TABLE III.

Uranyl Acetate in Water —185° C.

Concentration.	Character of Bands.							
	Broad.	Narrow.	Broad.	Narrow.	Broad.	Narrow.	Broad.	Narrow.
1 : 000	.5641μ		.5398μ		.5145μ		.4936μ	
1 : 160	.5640	.5533μ	.5398	.5284μ	.5152	.5048μ	.4931	.4837μ

	Very Broad.	Very Broad.	Very Broad.
1 : 1600	.5419μ	.5213μ	.5086μ
1 : 16000		.5239	.5024

4. *Uranyl Potassium Sulphate.*—Only a cursory examination of this substance was made. The concentrated aqueous solution when frozen

and excited showed broad bright bands without any trace of resolution at the temperature of liquid air. Diluted successively to 1/10, 1/100, 1/1000, and 1/10000 the spectrum retained the same appearance but the bands became broader as the concentration diminished and with the higher dilutions much dimmer.

B. Alcoholic Solutions.

Some of the uranyl salts, notably the acetate and nitrate, are soluble in ethyl alcohol. The concentrated alcoholic solution of the nitrate when reduced to the temperature of liquid air has a fluorescence spectrum consisting of alternately broad and narrow bands located as shown in Table IV.

The wave-lengths of the narrow bands is approximately the same as in the case of the aqueous solution of the nitrate but the broad bands are greatly shifted towards the red. In dilute solutions the pairs of bands are replaced by single broad bands having intermediate positions.

Table IV.

Uranyl Nitrate in Alcohol at —185° C.

Conc.	Broad.	Narrow.	Broad.	Narrow.	Broad.	Narrow.	Broad.	Narrow.
1/1	.5480	.5337	.5214	.5096	.5011	——	.4875	.4819
1/50	.5520	——	.5270	——	.5034	——	.4886	.4815?
1/500	.5522	——	.5271	——	.5040	——	.4882	.4821?
1/5000	.5521	——	.5273	——	.5041	——	——	.4806?

The narrow band at .4819 was vaguely discernible at higher dilutions but the position as indicated in the table was in question. The location of the bands of the dilute solutions does not seem to be very simply related to that of the bands of the concentrated solution although it remains nearly or quite unchanged for dilutions between 1/50 and 1/5000. This is easily explained if we consider these bands as resulting from the merging of neighboring bands of the concentrated solution and that the relative brightness of neighboring bands in the spectrum of the concentrated solution differs as we pass from red to violet.

The spectrum of the frozen alcoholic solution of uranyl acetate was altogether different from the various spectra already considered. It consisted, both for concentrated and dilute solutions, of numerous very narrow line-like bands superimposed upon an unresolved back ground; as though in this case a portion of the dissolved acetate had been thrown out on freezing.

The sharply resolved bands of this spectrum however, although they fall

into equally spaced series, as is the case with the spectra of the solid uranyl salts at low temperatures, do not correspond in position, nor as to interval, with the bands of the solid acetate.

In Fig. 2 the arrangement and approximate width of the various

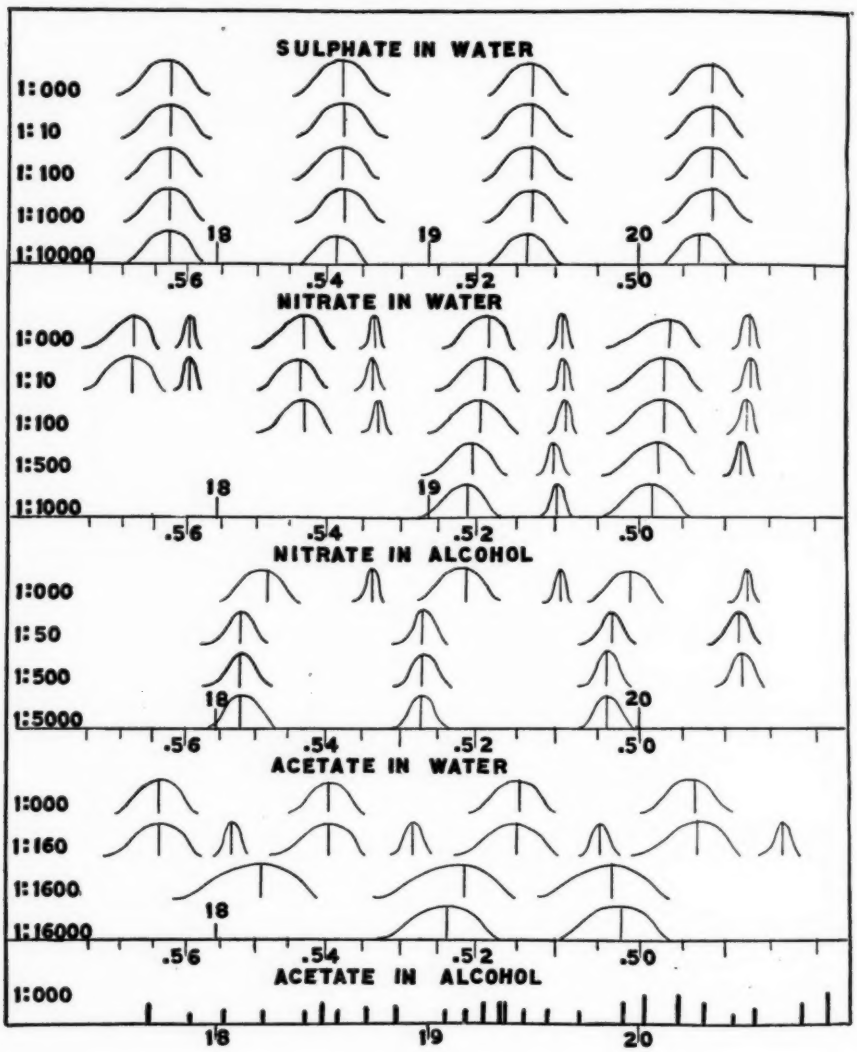

Fig. 2.

fluorescence bands already described are shown graphically in a diagram in which the abscissæ are frequencies. A scale of wave-lengths is also given for each spectrum. The positions of the narrow line-like bands in the spectrum of the acetate in alcohol are indicated at the bottom of the figure.

It will be seen that the frequency-interval between neighboring broad bands and also between narrow bands is approximately the same in all of these spectra; the average interval being about 86×10^{-4} where the wave-length is expressed in Ångström units. The only serious exception is in the case of the dilute solution of uranyl acetate in water (1 : 1600) where very broad bands have replaced the doublets of the more concentrated solution (1 : 160).

In the course of the observations recorded in this note, equally striking changes and shifts were found to occur when the temperature of a given solution was varied. A description of these phenomena may be expected as the result of a systematic study now being made by Mr. H. L. Howes, who has assisted the authors throughout this preliminary survey of the subject.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY,
March 30, 1914.

AN ATTEMPT AT AN ELECTROMAGNETIC EMISSION THEORY OF LIGHT.

BY JAKOB KUNZ.

THE principle of relativity gives a consistent explanation of the phenomena of aberration of light, of the experiments of Fizeau and Michelson-Morley, and of the increasing mass of the electron as function of the velocity. The new principle rejects the ether, in which according to the older theory light waves are propagated and in which the electric and the magnetic energies have their seat. We are concerned again with actions at a distance, without a medium, but with actions proceeding with the velocity of light.

The mathematical simplicity of the original principle of relativity was mainly due to the fact that it used a fundamental constant, the velocity of light c as an absolute constant, so that the Lorentz transformation can be applied to Maxwell's equations, which remain unchanged.

Recently A. Einstein[1] generalized the original principle and applying it to the field of gravity came to the conclusion that c must not be considered as a constant but as a function of the coördinates. If the conclusion of this investigation is confirmed by the experiment, then the original theory of relativity fails and if it is not confirmed, the theory of relativity will be beset with great difficulties. In either case it will only be an approximation to the physical reality.

If we consider the material bodies as completely separated but exerting forces on each other, then the action at a distance remains incomprehensible at all events; but if there is no medium, we should expect in accordance with the Newtonian theory of gravity an action at a distance with infinite velocity, and as a matter of fact we do not know whether gravity proceeds with finite or infinite velocity. If however in the theories of relativity it is assumed that the action proceeds with constant or variable finite velocity, then the phenomena become even more mysterious.

The principle of relativity, even in its simple original form, affects our

[1] A. Einstein, "Entwurf einer verallgemeinerten Relativitätstheorie," Zeitschrift für Mathematik und Physik, Band 62, p. 225, 1914.

notions of space and time. Time, once absolute, dwindles to a mere shadow. The simultaneity of two events and the equality of two time intervals become relative, the parallelogram of velocities appears only as an approximation, an absolutely solid body is impossible and the mass of a body depends on its velocity.

When a physical theory which is mathematically complicated and is only an approximation cuts so deeply in our fundamental notions, and renders the phenomena so incomprehensible, the freedom of advancing other theories, which, though more conservative, attempt to coördinate the various phenomena in question should be granted. In the following a theory will be developed which agrees with that of relativity in many features, but gives an entirely different aspect of the world.

§ 1. FUNDAMENTAL ASSUMPTIONS.

1. One of the theories other than that of relativity is the electromagnetic emission theory of light. It is a compromise between the emission theory and the wave theory. Each electric charge is supposed to be surrounded by an electromagnetic field residing in the medium, which field itself forms the mass of the charge. Thus instead of having a continuous medium, ether, we have as many media as there are electric charges. Each individual electromagnetic field extends throughout the universe, but is essentially concentrated in the immediate neighborhood of the electron. No assumption is made as to the structure of the elementary medium.

2. Maxwell's equations will be applied to the molecular fields. The expressions for the masses of fields at rest will be extended to fields in motion.

3. The velocity of light is always equal to c for a vacuum. While in a mechanical emission theory the velocity v of the source is added geometrically to the velocity c, we have in the present theory, through a process of compensation, the velocity of light always equal to c, and independent of the velocity of the source. The difference between a mechanical emission theory, the undulatory theory and the electromagnetic emission theory of light can be illustrated by the following figures.

The source of light moves with the velocity v per second from A to B towards the observer. In the mechanical emission theory the light particles emitted in the point A with the velocity c would lie after a second on the sphere with radius c and with the center B. Thus the center of the wave front would always coincide with the source itself. In the undulatory theory, where the light is carried through the continuous

independent medium, the center of the disturbance would always coincide with the point A in which it has been emitted. In the electromagnetic emission theory the center of the disturbance would coincide with the moving source but the wave surface would be an ellipsoid of revolution whose equatorial plane is perpendicular to the direction of

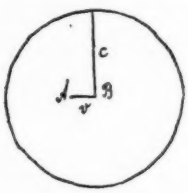

Fig. 1.

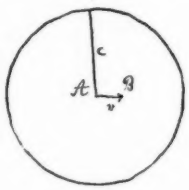

Fig. 2.

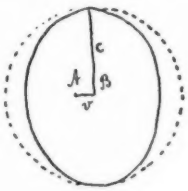

Fig. 3.

motion. In the second and third cases the velocity of light is always equal to c. In the second case the motion of the material luminous source has an influence on the optical phenomena, so we could hope to discover the motion of the source with respect to the ether and if the ether were at rest we could hope to discover the absolute motion of the source. This is impossible by mechanical methods according to New-

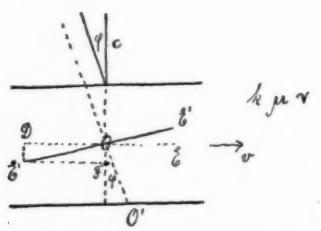

Fig. 4.

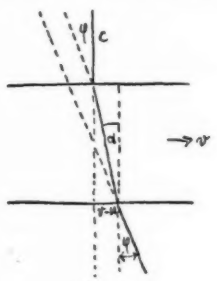

Fig. 5.

ton's principle of relativity. In the first and third theory we could not discover the absolute motion of the source. The critical velocity c of light in the vacuum is in Maxwell's theory equal to the ratio of the electrostatic to the electromagnetic unit of electric charge. If we consider c as constant and maintain Maxwell's equations unchanged for an electromagnetic field in motion, we consider that ratio of the two units also as independent of the motion. This means that the ratio of the force which on the one hand unit charge exerts upon another charge in a given distance to the force which on the other hand the same unit charge, when in motion exerts upon a magnet is independent of a uniform motion of the whole system. It is sufficient, but not necessary, for this purpose to assume that an electric charge

exerts upon another charge the same force, no matter whether both are at rest or in uniform motion; further, that an electric current exerts the same influence upon a magnet independent of the state of rest or of uniform motion of the whole system. In this way may be explained the facts that the electrostatic field of the earth, revolving round the sun, produces no magnetic effects, and the magnetic field of the earth no electric effects by electromagnetic induction upon bodies which are rigidly connected with the earth.

4. As there is no independent medium like ether, we are only concerned with relative motions between charges, magnets, sources of light and observers. An absolute motion of an electromagnetic system with constant velocity in a straight line can not be defined nor measured with optical and electrical methods.

The third and fourth assumptions lead to the Lorentz transformation of Maxwell's equations. There is however another transformation carried out by Maxwell and Hertz who found that the essential form of the equations remains unchanged if they are related to a system of axes at rest with respect to the ether or in motion similar to that of a rigid body; in other words, the absolute translation or rotation of a rigid system of bodies has no influence upon its internal electromagnetic phenomena, provided that all bodies of the system, the atomic fields included, take part in the motion. The electric and magnetic fields seem to be rigidly connected with the material bodies. The laws of geometrical optics are therefore independent of the motion of the earth.

The question still arises why according to this theory we can only discover relative motions between charges or magnets and between light sources and observers. In the first examples of course the reason lies in the interaction of the fields, but why should the field around a source of light contract in the equatorial plane if it approaches an observer? The reason may lie in the pressure which the light exerts upon the observer and which the observer exerts on the source. It might finally be possible that all the fields with which we can carry our experiments are imbedded as it were in a universal field of force.

§ 2. The Mass of the Electron.

An electron moves slowly in a medium whose permeability and dielectric constant are equal to unity. It is accompanied by a material electric field which, for small velocities, is symmetrical round about the spherical electron so that in a distance v from the center the electric force E is equal to $E = e/r^2$ and the magnetic force is equal to $H = ev \sin \vartheta/r^2 = Ev \sin \vartheta$; the magnetic energy per unit volume is equal to

$$E_1 = \frac{\mu H^2}{8\pi} = \frac{\mu E^2 v^2 \sin^2 \vartheta}{8\pi} = \frac{1}{2} m_1 v^2.$$

m_1 is the mass per unit volume.

$$m_1 = E^2 n' n^2 / 4\pi = \frac{\mu E^2 \sin^2 \vartheta}{4\pi}$$

or for $\mu = 1$

$$m_1 = \frac{E^2 \sin^2 \vartheta}{4\pi} .$$

in general

$$m_1 = \frac{\mu k^2 E^2 \sin^2 \vartheta}{4\pi} .$$

μ is the permeability and k the dielectric constant. For the following considerations it will be sufficient to put μ and k equal to 1. The mass dm of an infinitesimal ring will be equal to:

$$dm = \frac{e^2}{4\pi r^4} \sin^2 \vartheta \, 2\pi r^2 dr \sin \vartheta \, d\vartheta$$

and the whole mass will be equal to:

$$m = \frac{2}{3} \frac{e2}{a} = m_0,$$

where a is the radius, e the charge of the electron. This mass extends for an isolated electron throughout the whole space, but half of the mass is concentrated in the immediate neighborhood of the electron, that is in a sphere whose radius $a_1 = 2a$.

If the electron moves with finite velocity, then the electric field changes in such a way that the lines of electric force rotate towards the equatorial plane, which is perpendicular to the direction of motion v. At the same time the lines of magnetic force accumulate more and more in that plane as the velocity v increases. If finally the critical velocity c is reached, the whole electromagnetic field will be concentrated in that plane and the mass of the electron will increase indefinitely, so that an electric charge can not move with a velocity greater than that of light. We see also that in this limiting case the electron must cease to emit light in the direction of motion.

For a velocity v smaller than c we have:

$$E = \frac{e\left(1 - \frac{v^2}{c^2}\right)}{r^2 \left(1 - \frac{v^2}{c^2} \sin^2 \vartheta\right)^{\frac{3}{2}}},$$

$$dm = \frac{e^2 \left(1 - \frac{v^2}{c^2}\right)^2 \sin^2 \vartheta}{4\pi r^4 \left(1 - \frac{v^2}{c^2} \sin^2 \vartheta\right)^3} 2\pi r^2 dr \sin \vartheta \, d\vartheta,$$

whence

$$m = \frac{1}{2}e^2\left(1 - \frac{v^2}{c^2}\right)^2 \int\int \frac{dr}{r^2} \frac{\sin^3\vartheta\, d\vartheta}{\left(1 - \frac{v^2}{c^2}\sin^2\vartheta\right)^3}.$$

The integrations are to be extended over the whole field outside the electron. We do not know the shape of the electron, be it at rest or in motion. But there is a tension in the direction of electrical lines of force, and hence a resultant tension acting on the electron, especially round about the equator and the electron will assume the shape of an ellipsoid of revolution. According to the law which governs the equilibrium between internal and external forces, the mass as function of the velocity will be different. The integration will be carried out for three different conditions as follows:

1. The electron preserves the shape of a sphere during the motion.[1] The result of the integration is this:

$$\frac{m}{m_0} = \frac{3}{16}\left\{\frac{c^2 - v^2}{v^2} + 3 + \frac{c^2}{v^2}\left[\frac{3v}{(c^2 - v^2)^{\frac{1}{2}}} - \frac{(c^2 - v^2)^{\frac{1}{2}}}{v}\right]\operatorname{arctg}\frac{v}{\sqrt{c^2 - v^2}}\right\}.$$

2. The form of the electron changes according to the law

$$\frac{a}{b} = \sqrt{1 - \frac{v^2}{c^2}},$$

the integration yields the result

$$\frac{m}{m_0} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

the expression which relativity gives for the transversal mass of the electron.

3. The electron changes according to:

$$\frac{b}{a} = 1 - \frac{v^2}{c^2};$$

the integration gives

$$\frac{m}{m_0} = \frac{3}{8}\frac{c}{v}\frac{c^2 - v^2}{v^2}\left(\frac{c^2}{c^2 - v^2} - \frac{3}{4}\right)\log\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} + \frac{3c^4\left[2 - 3\left(1 - \frac{v^2}{c^2}\right)\right]}{16(c^2 - v^2)v^2}.$$

The first formula gives results which are smaller by 1 . . . 3 per cent. than the experimental values of C. E. Guye and S. Ratnowsky, which are however a little larger than those calculated by means of Abraham's

[1] J. Kunz, "Détermination théorique de la variation de la masse de l'électron en fonction de la vitesse," Archives des sciences physiques et naturelles de Genève, 1913.

formula. The third formula gives values too large and increasing too rapidly, while the second formula corresponding to relativity is in best agreement with the facts observed.

§ 3. The Electromagnetic Momentum and the Pressure of a Beam of Light.

It follows from Maxwell's equations that there is a tension in the direction of the lines of force, which per unit area perpendicular to the line is equal to the density of the energy. The pressure perpendicular to the lines of force is just as large. It follows that the pressure of a beam of light per unit area is equal to the electromagnetic energy per unit volume. We can now determine this pressure by means of the electromagnetic mass and momentum. A beam of light consists in the present theory of oscillating and advancing electromagnetic mass. The electric force is perpendicular to the direction of propagation, $\sin \vartheta = 1$ and if $\mu = k = 1$, then

$$m_1 = \frac{E^2}{4\pi}.$$

The momentum per unit volume is equal to

$$M = m_1 c,$$

the energy per unit volume will be

$$\frac{1}{2} m_1 c^2 = \frac{E^2 c^2}{8\pi} = \frac{H^2}{8\pi}$$

and the pressure per unit area is equal to

$$p = Mc = \frac{H^2}{4\pi}.$$

If

$$H = H_a \cos \frac{2\pi}{T}\left(t - \frac{x}{c}\right)$$

then

$$\bar{H}^2 = \frac{1}{2} H_a^2$$

and

$$p = Mc = \frac{1}{8\pi} H_a^2;$$

this is the energy of the beam per unit volume.

If k and μ are both equal to 1, then

$$p = m_1 c^2 = E, \quad m = m_1 = \frac{E}{c^2},$$

or by differentiation

$$dm = \frac{dE}{c^2}.$$

Hence it follows that a source of radiation, which emits energy, loses a part of its electromagnetic mass. The sun loses yearly about 10^{14} tons of electromagnetic inertia. On the other hand if a body absorbs energy, its mass must increase proportionally to the energy absorbed, and if an electric charge is set in motion, it will have more magnetic energy than at rest. If this electromagnetic mass were granular and could be broken up into smaller units, such as $E = hn$, then such a unit would have the mass for yellow light $m_1 = 3_1 F.10^{-33}$, about 100,000 times smaller than the mass of the electron at rest.

§ 4. On Newton's Dynamical Equations.

Every atom possesses at least one electron. If the velocity of an atom changes, the inertia will change also. Newton's dynamical equations require therefore a correction which for all ordinary velocities of material ponderable bodies is insignificant, but which becomes very large, if the velocity v approaches that of light. The law of conservation of mass does not hold rigorously, but the law of conservation of momentum remains exact.

The total momentum remains constant in an enclosed system of heavy bodies, electrical charges, magnets, currents and sources of light. If a source emits a beam in a definite direction, it will lose momentum and be driven in the opposite direction. If on the other hand an electric wave strikes a charge, or if a beam is absorbed by a surface, then the material bodies gain as much momentum as disappears from the space. A force is defined in Newton's dynamics by the following equation:

$$F = \frac{dM}{dt},$$

but since

$$\frac{dM}{dt} = \frac{d(mv)}{dt},$$

$$F = \frac{mdv}{dt} + \frac{vdm}{dt}$$

and

$$Fdt = m\,dv + v\,dm.$$

If the mass moves through space dl during time dt, then the increase of energy is equal to:

$$dE = F\,dl = F\frac{dl}{dt}dt = Fvdt = dmv^2 + mvdv = c^2dm,$$

$$dm(c^2 - v^2) = mv\,dv,$$

$$dm\left(1 - \frac{v^2}{c^2}\right) = \frac{m}{c^2}v\,dv.$$

This equation has been integrated by Lewis and Tolman. Putting $v/c = x$, we get:

$$\frac{dm}{m} = -\frac{1}{2}\frac{d(1-x^2)}{1-x^2},$$

$$\log m = \log (1-x^2)^{-\frac{1}{2}} + \log m_0,$$

$$\frac{m}{m_0} = \frac{1}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}};$$

hence we find again for the increase of the mass the expression given by relativity.

The corrected equation of Newton holds not only for the ordinary inert bodies, but also for the radiations in a cavity. In a cavity bounded by perfect mirrors, we may find for the radiant energy E, the expression

$$E = mc^2,$$

or the energy of radiation possesses inertia. If moreover this electromagnetic inertia is subject to gravity, then the weight of such a cavity will be equal to:

$$mg = \frac{Eg}{c^2}.$$

If further the electromagnetic mass is at the same time heavy, then the gravity of the earth will exert on a certain body a force in a given point, which depends on the state of motion or rest of the body. An ordinary potential of gravity, as a function of the coördinates, only exists no more, for it now depends on the velocity of the falling body as well.

If the electromagnetic mass is subject to gravity, then a beam of light from a fixed star, passing through the field of attraction of the sun, will be attracted and therefore the position of the star will appear displaced. This very important problem may be solved by this phenomenon or also by observations made with pendulums of radioactive substances which are very rich in electrons. Let us consider two geometrically similar pendulums, the first consisting of a radioactive substance, such as radium, the second of non-radioactive substance. We shall assume the weight Mg of the two pendulums to be the same, but the mass M of the radioactive substance to be $m_1 + m$, where m_1 shall be subject to gravity, the electromagnetic mass m independent of gravity. The periods of the two pendulums will be

$$T_1 = 2\pi\frac{\sqrt{(m_1+m)r^2}}{Mgs},$$

$$T_2 = 2\pi\frac{\sqrt{m_1 r^2}}{Mgs}.$$

The radioactive pendulum would have a longer period than the ordinary one. 1 gr. radium contains about 1/13 mgr. more mass in the active state than after the transformations. In recent years it has been shown by Eötvös that for ordinary bodies the inertia is exactly proportional to the weight up to 10^{-7}. But nevertheless we have not yet a direct experimental proof that the electromagnetic mass is subject to gravity.

§ 5. The Experiment of Michelson-Morley.

As there is no independent medium, the motion of the earth has no influence upon geometrical optics and the result will remain the same whether we place the interference apparatus of Michelson-Morley in the direction of the motion of the earth or perpendicular to it. Even if the source of light were not connected with the apparatus, but were in motion, as for instance if the light of canal rays were made use of or the light of a star, in no case would we observe a displacement of the interference fringes through a rotation of the apparatus. Here appears a distinct difference between the electromagnetic and the mechanical emission theories. According to the latter theory we should expect an effect in the experiment of Michelson-Morley, if the light were incident from a star.

§ 6. The Experiment of Tronton and Noble.

The energy of an electric condenser of two plane parallel plates is independent of the direction of the motion of the earth; this experimental fact follows immediately from our assumptions. In the theory of an independent ether however the condenser would possess more energy if the plates were parallel to the velocity v of the earth, than if they were perpendicular to it. A charged and suspended condenser would produce a couple in the first position tending to bring it into the second position.

§ 7. Aberration of the Light from Fixed Stars.

While the light of a fixed star travels from the objective A of the telescope to O', the earth moves with the velocity v from O to O'. The phenomenon of aberration was always evidence in favor of an emission theory or led to the assumption of a stationary ether, through which the earth moves.

$$\frac{OO'}{O'A} = \frac{v}{c} = \frac{\sin \Phi}{\sin \vartheta} = 20'' 49.$$

Φ is the angle of aberration, v/c the constant of aberration of the light from fixed stars.

§ 8. THE EXPERIMENTS OF AIRY AND FIZEAU.

As the constant of aberration v/c depends only on v and c, Airy thought that it must change, if c changes. He filled therefore the telescope with water and expected a different angle of aberration, as the velocity of light in water is equal to c/r, if r is the index of refraction of water. Airy found however no change of the constant of aberration and he concluded that the water carries the ether with it, so that the velocity v is diminished by the same measure as c. If the water were carrying the ether with it with its own velocity, then no aberration would be possible, it must therefore communicate to the ether only a fraction of its own velocity. If the oscillating and advancing mass of a beam of light falls upon a transparent substance containing bound electrons, these charges will be set in motion and emit electromagnetic mass themselves. If moreover the substance struck by light is in motion, the electrons will be deviated from their original direction and oscillate in a new path. The light emitted will be perpendicular to this new direction and the original beam of light appears to be deflected from the original direction.

A beam of light strikes a column of water with plane surfaces, which move with constant velocity v perpendicular to the beam of light. Let us consider in a given point O of Fig. 4 an electron, which, if the water is at rest, under the action of the electric force OE, is deflected in the direction OD. The magnetic force would have no influence. If however the electron together with the water is set in motion with the velocity v, then the magnetic field of the light will act upon the charge in motion tending to deflect it in the direction OF. The resulting deflection and oscillation will be along OE'; the new beam will travel in a path perpendicular to this direction, that is from O to O'.

$$OD = eE,$$

$$OF = \frac{evH}{c^2} \qquad H = cE,$$

$$OF = \frac{evE}{c},$$

$$\sin \Phi = \frac{OF}{OD} = \frac{v}{c};$$

this means that the angle of aberration Φ is independent of the specific properties k and r of the medium. Hence Fizeau's experiment follows immediately.

The water communicates to the beam a part of its own velocity v, so that the beam travels in the direction of v with a velocity u. It will

strike a point A on the lower side of the layer of water, and be deflected, so that Φ represents again the angle of deviation between the real and the observed beam. Now we have

$$\sin \alpha = \frac{v - u}{V},$$

$$\frac{\sin \Phi}{\sin \alpha} = r, \quad c = Vr,$$

$$\sin \Phi = r \sin \alpha = \frac{r(v - u)}{V} = \frac{r^2(v - u)}{c}.$$

This angle however is independent of the specific properties of the flowing substance; hence for the vacuum:

$$r = 1, \quad u = 0,$$

$$\sin \Phi = \frac{v}{c},$$

$$\frac{v}{c} = \frac{r^2}{c}(v - u), \quad (v - u)r^2 = v,$$

$$u = v\left(1 - \frac{1}{r^2}\right).$$

This is according to Fresnel and Fizeau the fraction of the motion, which the flowing water communicates to the beam of light.

If we observe a point at rest through a rotating disc of glass, it will appear to be deflected from its natural position. If we use a Roentgen ray instead of a beam of light then r the index of refraction is equal to 1 and therefore $u = 0$, that is, we would expect that Fizeau's experiment gives a negative result with Roentgen rays.

Recent Literature.

The present attempt at an electromagnetic emission theory is based upon the works of Faraday, Maxwell, H. A. Lorentz and other investigators. J. J. Thomson especially has in various investigations treated the electromagnetic field of an elementary charge as something individual, endowed with mass, momentum and energy. He has however, so far as I know, not extended the theory to the critical phenomena here treated. Contributions to the present theory have been made by N. R. Campbell in his book on modern electrical theory, by D. Comstock, Physical Review, 30, p. 267, 1910; R. C. Tolman, Physical Review, 31, p. 26, 1910; 35, p. 136, 1912; O. M. Stewart, Physical Review, 32, p. 418, 1911, and J. Kunz, American Journal of Science, 30, p. 313, 1910.

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March 12, 1914.

NOTE ON THE EVALUATION OF THE CONSTANT C_2 IN PLANCK'S RADIATION EQUATION.

BY CHARLES N. HASKINS.

THE methods hitherto used[1] for the evaluation of C_2 in Planck's equation

$$J = \frac{c_1}{\lambda^5} \frac{1}{e^{C_2/\lambda\theta} - 1} \tag{1}$$

depend essentially upon graphical processes, *i. e.*, upon processes whose precision is limited by conditions entirely independent of those which control the accuracy of the experimental data. It seems therefore worth while to point out a purely numerical method by means of which C_2 may be found from the experimental results without the necessity of recourse to such graphical methods.

In the present note such a method is developed. The problem is reduced to the solution of an equation of the form

$$u^n - u + A = 0,$$

where n is a positive quantity greater than unity but not necessarily an integer. This equation may be solved by any one of several well-known methods. The details of one of these methods are, for completeness, indicated, and the applicability of the process tested by applying it to data computed by means of equation (1). It is found that the assumed value of C_2 is reproduced with all the precision of which the logarithmic tables used are capable.

Reduction of the Equation.—Let J_1, λ_1, θ_1; J_2, λ_2, θ_2 be two sets of observed corresponding values of J, λ and θ. Let $\lambda_2\theta_2 > \lambda_1\theta_1$. Then

$$J_1 = \frac{c_1}{\lambda_1^5} \frac{1}{e^{C_2/\lambda_1\theta_1} - 1}, \quad J_2 = \frac{c_1}{\lambda_1^5} \frac{1}{e^{C_2/\lambda_2\theta_2} - 1}. \tag{2}$$

Hence

$$\frac{e^{C_2/\lambda_1\theta_1} - 1}{e^{C_2/\lambda_2\theta_2} - 1} = \frac{J_2\lambda_2^5}{J_1\lambda_1^5}. \tag{3}$$

Put now

$$n = \frac{\lambda_2\theta_2}{\lambda_1\theta_1},$$

[1] Cf. Buckingham and Dellinger, Bulletin, Bureau of Standards, 7, 393–406, 1911.

$$x = e^{C_2/\lambda_2\theta_2},$$

$$C = \frac{J_2\lambda_2^5}{J_1\lambda_1^5}.$$

Then since $\lambda_2\theta_2 > \lambda_1\theta_1 > 0$, and $C_2 > 0$; $n > 1$, $x > 1$, $C > 1$. Equation (3) now becomes

$$\frac{x^n - 1}{x - 1} = C,$$

or

$$x^n - Cx + C - 1 = 0. \tag{4}$$

This is an equation of the well-known "trinomial form" and may be solved, for example, by Newton's Method, by the method of Gaussian logarithms, or by iteration.[1]

If we put further

$$u = \frac{x}{C^{1/n-1}} = \frac{e^{C_2/\lambda_2\theta_2}}{C^{1/n-1}},$$

whence

$$C_2 = \frac{\lambda_2\theta_2}{\log_{10} e}\left(\frac{1}{n-1}\log_{10} C + \log_{10} u\right), \tag{5}$$

and

$$A = \frac{C-1}{C^{n/n-1}}, \quad 0 < A < 1,$$

equation (4) becomes

$$u^n - u + A = 0. \tag{6}$$

Equation (4) has two and only two positive roots. One is the trivial value $x = 1$ corresponding to $C_2 = 0$, the other is greater than unity and corresponds to the value of C_2 sought. Hence equation (6) has two positive roots of which the greater gives the desired value of C_2. It is easily seen that both roots of (6) lie between zero and unity. Write now equation (6) in the form

$$u = (u - A)^{1/n}. \tag{7}$$

If we consider the curves

$$y = u, \quad z = (u - A)^{1/n}$$

and note that $y(A) = A > z(A) = 0$, while $y(1) = 1 > z(1) = (1 - A)^{1/n}$ we see that in the neighborhood of the greater root of (6)

$$1 = \frac{dy}{du} > \frac{dz}{du} > 0.$$

Hence[2] if by substitution in (6) we find a rough approximation u_0

[1] Cf. Runge, Praxis der Gleichungen, pp. 52, 141, 81; and especially p. 147, where an equation of form (4) arising from a financial problem is solved by the method of Gaussian logarithms.

[2] Cf. Osgood, Calculus, p. 403.

to the greater root the successive approximations given by

$$u_1 = (u_0 - A)^{1/n},$$
$$u_2 = (u_1 - A)^{1/n},$$
$$\cdots\cdots\cdots\cdots$$
$$u_{m+1} = (u_m - A)^{1/n},$$

will converge to the desired root u. When this value has been found C_2 is given by

$$C_2 = \frac{\lambda_2\theta_2}{\log_{10} e}\left(\frac{1}{n-1}\log_{10} C + \log_{10} u\right).$$

Numerical Example of the Method.—A test of the above process *as a method of computation* is best obtained by applying it to values of J computed from (1) with assumed values of C_1 and C_2 and noting the precision and rapidity with which the assumed value of C_2 is reproduced.

The following data were assumed:

$$C_1 = 5.29 \times 10^5,\ C_2 = 1.46 \times 10^4,\ \lambda_1 = 2,\ \theta_1 = 800,$$

$\lambda_2 = 5$, $\theta_2 = 1{,}600$ and it was found that

$$J_1 = 1.801,\ J_2 = 32.54,\ u = 5,\ C = 1764,\ A = 0.1542$$

The equation to be solved is therefore

$$f(u) = u^5 - u - 0.1542 \quad 0.$$

Substitution gives $f(0.9) = -0.155$, $f(1) = A = 0.154$. Hence the root sought is approximately $u_0 = 0.95$.

The following table gives the result of the successive approximations.

m	u_m	C
0	0.95	1.454×10^4
1	0.9554	1.459×10^4
2	0.9565	1.460×10^4
3	0.9570	1.460×10^4
4	0.9570	

The computations were made with four place logarithmic tables. It is seen that the convergence of the process is completed with $u_4 = u_3$ but that u_1 approximately and u_2 exactly reproduces the assumed value of C_2.

Though the precision of experimental data would not justify the use of seven place tables, it was deemed desirable to test the *method* by a computation with such tables. The convergence of the u's (to seven places) was complete at $u_9 = u_8$, but u_6 gave $C_2 = 1.459999 \times 10^4$, and u_7, u_8, u_9, gave $C_2 = 1.460000 \times 10^4$.

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March 29, 1914.

PROPOSED INVESTIGATION OF THE EFFECT ON THE PROPAGATION OF ELECTRIC WAVES OF THE TOTAL ECLIPSE OF THE SUN, 21st AUGUST, 1914.[1]

THE forthcoming total eclipse of the sun affords an exceptional and important opportunity of adding to existing knowledge of the propagation of electric waves through air in sunlight and in darkness, and across the boundaries of illuminated and unilluminated regions. The eclipse will be total along a strip extending from Greenland across Norway, Sweden, Russia and Persia to the mouths of the Indus. In Russia the duration of totality will be a little more than two minutes.

There are two main points calling for investigation during the eclipse. In the first place, the propagation of signal-bearing waves through air in the umbra and penumbra will probably obey laws different as regards absorption and refraction from those obeyed in illuminated air. In the second place, the strength, frequency and character of natural electric waves, and of atmospheric discharges, may vary. The variations may occur either because the propagation of natural waves from distant sources is facilitated or impeded by the eclipse, or, possibly, because the production of natural electric waves or atmospheric discharges is for some unknown reason affected by the eclipse.

These points have previously been investigated to only a slight extent. The observers of signals during the solar eclipse of 17th April, 1912, nearly all agreed that the strength of the signals was greater during the eclipse than an hour before or after. There was only one special observation of strays during the same eclipse, when very pronounced and remarkable variations were recorded during the passage of the shadow-cone across Europe.

To investigate the propagation of signals across the umbra it will be necessary to arrange for wireless telegraph stations on either side of the central line of the eclipse to transmit signals at intervals while the umbra passes between them. This transit of the umbra occupies about two minutes. It is thus very desirable that the Scandinavian and Russian stations should transmit frequently throughout several minutes before, during, and after totality. But stations other than those favored by their proximity to the central line should endeavor to keep a complete record of the variations of signals during the eclipse. Stations in Europe west of the central line and stations in the Mediterranean and in Asia Minor may find noticeable changes in the strength of signals, particularly long distance signals, between the hours of 10 a.m. and

[1] Received from the Committee for Radiotelegraphic Investigation of the British Association for the Advancement of Science.

3 p.m., Greenwich time; and it is probable that the stations of India and East Africa, and ships in the Indian Ocean, may feel the effect of the penumbra in the afternoon. On the other hand, ships in the Atlantic, and fixed stations in eastern Canada and the United States, will probably be affected by the penumbra in the early morning. At Montreal the eclipse (partial) is at its greatest phase at 5.52 a.m. Standard time. It is possible that the eclipse may have some influence even when it is invisible.

The investigation of strays is of as great interest as that of signals. So far as is yet known, the natural electric waves reaching wireless telegraph stations in latitudes higher than 50° appear to travel mostly from the south. Thus the greatest changes produced in strays by the eclipse will probably be experienced at stations in Scandinavia and Russia, to reach which the waves must cross the path of the umbra. At the same time changes of some kind are to be expected in other districts than these, and it is therefore desirable that statistical observations of natural electric waves be made all over the world, and especially at places within an earth quadrant of southern Russia. It is also desirable that meteorological observations, including those of atmospheric ionization and potential gradient, should be at the disposal of the Committee when considering the records of strays and signals.

The Committee propose to prepare and circulate special forms for the collection of statistics of signals and strays, especially within the hemisphere likely to be affected by the eclipse; they will endeavor to make provision for the transmission of special signals at times to be indicated on the forms; and they will offer for the consideration of the authorities controlling stations near the central line a simple programme of work. The discussion of the observations, and the comparison with meteorological data, will be carried out by the Committee; and digests of the statistics, together with the conclusions drawn from the analysis, will be published in due course.

The Committee would be greatly aided in the organization of this investigation if those possessing the necessary facilities and willing to make observations during the eclipse would communicate with the Hon. Secretary, Dr. W. Eccles, University College, London, W. C., at the earliest possible date.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

WASHINGTON MEETING, APRIL 24 AND 25, 1914.

MINUTES OF THE SEVENTY-SECOND MEETING.

A REGULAR meeting of the Physical Society was held at the Bureau of Standards, Washington, April 24 and 25, 1914. This was a joint meeting with the Electrophysics Committee of the American Institution of Electrical Engineers. The programs of the two morning and of the Saturday afternoon sessions were in charge of the Physical Society with President Merritt in the chair. The Friday evening session was in charge of the A. I. E. E. with Chairman J. B. Whitehead in the chair. The Friday afternoon session was given to a lecture by Sir Ernest Rutherford, F.R.S., of the University of Manchester, Eng., "On X-ray and Gamma-ray Spectra," complimentary to the American Physical Society.

Special features of the meeting were the opening of the new electrical building of the Bureau of Standards, and an exhibit of physical apparatus. This exhibit was arranged under the direction of a committee of the American Physical Society with Dr. F. A. Wolff as chairman. It was a large and representative exhibit, participated in by thirty-two manufacturers, importers and industrial research laboratores, ten universities and educational institutions, and eight federal scientific bureaus.

Members of the Physical Society were especially invited by the National Academy of Sciences to attend the William Ellery Hale lectures by Sir Ernest Rutherford upon "The Constitution of Matter and the Evolution of the Elements." These were given in the auditorium of the National Museum on April 21 and 23, and were attended by a large number of the Physical Society members.

All in attendance at the meetings were the guests of the scientific staff of the Bureau of Standards at luncheon on both days of the meeting. After the Friday evening session the local branch of the A. I. E. E. gave a smoker which was largely attended.

At a short business session of the Physical Society the following items of business were transacted:

On recommendation of the council, it was voted to establish a new grade

of foreign members, to be defined as non-residents of North America, to pay dues of $4 with no initiation fee and to receive the PHYSICAL REVIEW (with *Science Abstracts* on additional payment of $2) and having all rights of regular members in the Society. Also to make such changes in the by-laws as the establishment of this new grade of membership would necessitate.

On motion it was voted to approve and authorize an International Congress of Physics to be held in Washington in October, 1915, in case it should appear that it can be properly financed. (A committee of nine was appointed by the council to determine this question.)

It was voted that the president appoint a committee of three to express the deep sense of loss felt by the members of this Society in the death of their former president, Professor B. O. Peirce.

The Society voted to express to the director and members of the National Bureau of Standards its high appreciation of the generous hospitality extended to the Society throughout the meeting, also to the Washington Section of the American Association of Electrical Engineers for arranging trips and providing guides to various places of scientific interest in the city and its neighborhood.

The attendance at all sessions was exceptionally large—at none less than one hundred and fifty. The Friday afternoon lecture was heard by about three hundred. One hundred and fifty-seven members registered.

At the Friday evening session (in charge of the A. I. E. E.) the following papers were presented:

Solenoids. C. R. UNDERHILL.

Some Investigations of Lighting Protection for Buildings. DEBLOIS.

Some Simple Examples of Transmission Line Surges. (By title.) W. S. FRANKLIN.

At the three sessions in charge of the American Physical Society the following papers were presented:

High Temperature Measurements with the Stefan-Boltzmann Law. C. E. MENDENHALL and W. E. FORSYTHE.

Cold-end Compensator for Thermocouples. (Apparatus shown.) CHARLES B. THWING.

The Emissivity of Metals and Oxides. I; Nickel Oxide (NiO) in the Range 600° to 1,300° C. G. K. BURGESS and P. D. FOOTE.

Formulæ for the Ordinary Mercury Contact Thermostat, and Some Practical Conclusions Deduced from Them. (By title.) W. P. WHITE.

The Specific Heats of Mixtures of Water and Alcohol, and of Solutions of Non-Electrolytes in these Mixtures. W. F. MAGIE.

The Extension of the Spectrum in the Extreme Ultra-Violet. (By title.) THEODORE LYMAN.

The Infra-red Arc Spectrum of Barium. H. M. RANDALL.

On the Growth and Decay of Color Sensation. M. LUCKIESH. (Read by A. G. Worthing.)

Some Effects of Diffraction on Brightness Measurements Made with the

Holborn-Kurlbaum Optical Pyrometer. A. G. WORTHING and W. E. FORSYTHE.

Displacement of Arc Lines not Due to Pressure. CHAS. E. ST. JOHN and H. D. BABCOCK. (Read by C. G. Abbott.)

On the Accuracy of Terrestrial-Magnetic Measurements. L. A. BAUER.

The Gamma-Ray Comparison of Specimens of Radium Salts. N. ERNEST DORSEY.

The Results of the Atmospheric Electric Observations on the Second Cruise of the 'Carnegie,' June 1910 to December, 1913. C. W. HEWLETT.

Apparatus for the Spectroscopic Synthesis of Color. H. E. IVES and E. J. BRADY.

New Methods for Measuring Time Constants of Low Resistances. FRANK WENNER, ERNEST WEIBEL and F. B. SILSBEE.

A Sensitive Moving Coil Galvanometer. F. WENNER, E. WEIBEL and F. C. WEAVER.

Some Peculiarities in the Thermal Expansion of Invar. ARTHUR W. GRAY.

A New Turbidimeter. P. V. WELLS.

The Diurnal System of Convection. WM. H. BLAIR.

The Control of the Wave-length-Sensibility Curves for Selenium. E. O. DIETERICH.

Some Records of the Wireless Time Signals Made with a Physiological Recorder. C. W. WAGGONER.

Electric Conduction and Thermo-electric Action in Metals. E. H. HALL.

Electrochemical Indicators and Recorders. Instruments for Showing Continuously the Chemical Content of Solutions. F. A. HARVEY.

Characteristic Curves of Tungsten Filament Incandescent Lamps and their Application in Heterochromatic Precision Photometry. G. W. MIDDLEKAUFF and J. F. SKOGLAND.

The Thomson E.M.F. in and the Thermal Conductivity of Tungsten at Incandescent Temperatures. A. G. WORTHING.

A Direct Determination of "h." R. A. MILLIKAN.

Reversible Transitions Between Solids at High Pressures. P. W. BRIDGMAN.

Surface Leakage over Insulators. H. L. CURTIS.

Spark Potentials in a Magnetic Field. (By title.) R. F. EARHART.

Further Experiments on the Use of the Photo-Electric Cell in Stellar Photometry. JAKOB KUNZ, J. STEBBINS and W. F. SCHULZ.

Note on the Photo-Electric Effect with Potassium Surfaces in Very High Vacuum. SAUL DUSHMAN.

On the Relation Between the Photo-electric Potential and the Frequency of Light for Potassium. S. KARRER.

Reflection and Scattering of Slow-moving Electrons. ALBERT W. HULL.

Wave-Length Sensibility Curves of Potassium Photo-Electric Cells. H. E. IVES.

The Photo-Electric Effect of Carbon as Influenced by its Absorbed Gases. OTTO STUHLMANN, JR. and R. J. PIERSOL.

The Relation of Residual Gases to the Photo-Electric Sensitiveness and the Contact E.M.F. of Sodium. R. A. MILLIKAN and W. H. SOUDER.

Effect of Glass Walls on Thermionic Currents. SAUL DUSHMAN.

Corona Produced by Continuous Potentials. S. P. FARWELL.

Theory of the Corona. BERGEN DAVIS.

A Milli-Ampere Current Transformer. EDWARD BENNETT.

Some Points with Regard to the Variation of the Specific Magnetization of a Substance with Temperature. W. F. G. SWANN.

High Frequency Verification of Kirchoff's Capacity Formulæ. (By title.) F. C. BLAKE and CHAS. SHEARD.

An Electromagnetic Puzzle. F. J. ROGERS.

A Significant Instance of Galvanometer Instability. W. P. WHITE.

A New Design of Flicker Photometer for Laboratory Colored-Light Photometry. H. E. IVES and E. J. BRADY.

Note on the Physiological Effect of the Current. F. J. ROGERS.

Examples of the Precision Attainable in Determinations of Thermal Expansivity. ARTHUR W. GRAY.

Dioptric Formulæ for Combined Cylindrical Lenses at Oblique Axes. (By title.) CHARLES SHEARD.

The Testing of Potentiometers. FRANK WENNER and ERNEST WEIBEL.

An Absorbing Solution for Eliminating Color Differences in Photometry. H. E. IVES and E. F. KINGSBURY.

Photographs of Retrograde Rays, (*a*) from the Cold Cathode, (*b*) from the Hot Lime Cathode. O. H. SMITH.

A. D. COLE,
Secretary.

ON METALLIC CONDUCTION AND THERMOELECTRIC ACTION IN METALS.[1]

BY EDWIN H. HALL.

IN this paper[2] an attempt is made to consider and compare the functions of (*A*) electrons transferred from atom to atom during collisions or contacts and of (*B*) electrons relatively free between the atoms.

The proposition is advanced that thermoelectric action cannot be accounted for without assigning thermal capacity to the electrons, and the idea that such thermal capacity is a function of the temperature is introduced by treating R, of the equation $pv = RT$, as a variable for electrons, $= k_r T^{\rho}$, where k_r and ρ are constants.

The main conclusions are that the free electrons (*B*) are essential for the

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914.

[2] Acting upon information which I had not examined with sufficient care, I stated at Washington that pure metals, when heated under such pressure conditions as not to suffer change of volume, show a decrease of resistance. This statement is true regarding liquid mercury, as Barus found long ago, but it is not true of metals in the solid state. After going carefully over the ground of my investigation once more I find that the mistake which I had made in this particular did not affect my general conclusions, which remain unchanged.

phenomena of thermoelectricity, but play an unimportant part in electric conduction.

The relation between the Peltier effect and the Volta effect is discussed.

A table of data concerning change of volume and change of resistance in the fusion of metals is given in an Appendix.

CAMBRIDGE, MASS.,
May 26, 1914.

SOME POINTS WITH REGARD TO THE VARIATION OF THE SPECIFIC MAGNETIZATION OF A SUBSTANCE WITH TEMPERATURE.[1]

BY W. F. G. SWANN.

ACCORDING to the theory of Weiss the molecular magnetic theory of a solid may be treated on the lines of that for a gas provided that the applied field H is replaced by $H + H_m$, where H_m is a field which represents the effects of the molecular actions. H_m is assumed to be of the form NI where I is the intensity of magnetization and N is a constant for the substance.

Weiss has shown that in order to account for the variations of the specific magnetization with temperature above the Curie point, it is necessary to assume that the number of magnetons in a molecule changes abruptly at certain temperatures. In the present communication it is shown that this is not all that is necessary to account for the facts, but that the constant N must also vary abruptly with the temperature and in some cases must even acquire a negative value.

A change in the number of magnetons is usually not accompanied by an abrupt change in the coefficient of magnetization, which fact seems to indicate a tendency on the part of the substance to make a change in the number of magnetons, only when the temperature conditions are such that it can do so without altering that part of the energy which is purely magnetic.

DEPARTMENT OF TERRESTRIAL MAGNETISM,
CARNEGIE INSTITUTION OF WASHINGTON.

CHARACTERISTIC CURVES OF TUNGSTEN FILAMENT INCANDESCENT LAMPS AND THEIR APPLICATION IN HETEROCHROMAT C PRECISION PHOTOMETRY.[1]

BY G. W. MIDDLEKAUFF AND J. F. SKOGLAND.

AS the international candle is maintained by means of carbon filament incandescent lamps operated at four watts per candle, the standardization of lamps of higher efficiency involves photometric difficulties due to color difference. Although differences in color may be reduced or eliminated in the direct comparison of the lamps either by the use of blue glass screens placed in the path of the light from the carbon standards, or by the substitution of secondary standards of the same color as the lamps to be measured, nevertheless the difficulties mentioned must be encountered in calibrating the screens and the secondary standards.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914.

On the assumption that the radiating properties of pure tungsten wire in the form of a filament in vacuo are constant, the authors concluded that if the candle-power-voltage relation could be accurately expressed by means of an equation, candle-power values of tungsten standards at the higher efficiencies could be determined from single values at color match with carbon standards with greater precision by computation than by actual measurement with color difference. It would then be possible, in the calibration of glass screens and incandescent lamps at high efficiencies, to avoid all measurements with color differences except those necessary in the determination of the equation which would be carefully and accurately made once for all.

To investigate this subject, a group of 60-watt, 110-volt, drawn wire tungsten lamps were standardized through the range of color met with in the standardizing work of the Bureau of Standards, and it was found, as expected, that not only the candle-power-voltage relation but every other characteristic relation for each and every lamp of the group could be accurately expressed by a simple equation of the form

$$y = Ax^2 + Bx + C,$$

in which x = log voltage; y = log candlepower, log watts, log current, or log watts per candle; and A, B and C are constants whose numerical values depend upon the significance of y. C depends also upon the candle-power value of the lamp considered while the values of A and B are invariable for the same relation when applied to any lamp of the group investigated.

It was found that the equations generally employed to express characteristic relations could not be used except through a limited range and that the adjustment is in no case as precise as that obtained by means of the above equation.

As watts per candle is the only independent variable, it must be known before computing characteristic values. Therefore the general equations for the group of lamps had, necessarily, to be arranged for the substitution of percentage values with unity taken at some chosen efficiency. In this case 1.20 w.p.c. was found to be most practical. Volt ratios thus have their base at this point, and in the general equations which are also of the form given above, x = log per voltage and y = log per candlepower, log per watts, log per current, or log actual watts per candle. C is equal to zero except in the equation for watts per candle evaluation where it has the value log 1.20. To the constants A and B, values are assigned which give practically perfect adjustment of candle-power and watts of these lamps and also permit extrapolation to 0.7 w.p.c. and all intermediate points as has been verified by a check with independent groups of lamps. The range from 0.7 to 0.5 w.p.c. has not yet been investigated, but it seems probable that this further extension is possible. The equations have been found to apply to tungsten lamps of sizes from 20 to 100 watts, including those of most recent manufacture.

Derived equations permit the reduction of candle-power and voltage values at any efficiency to those at 1.20 watts per candle. This reduction is the first essential in computing characteristic values.

Tables will be published[1] showing, (*a*) values of per cent. candle-power and per cent. voltage at 1.20 watts per candle from any given efficiency within the range from 2.0 to 0.7 and probably to 0.5 in steps of 0.02 watts per candle, and (*b*) percentage values of watts and candle-power and actual watts per andle for every volt ratio from 0.6 to 1.3 and probably to 1.6. Tabular differences will be given and their use explained. From these tables others based on any efficiency included can be easily calculated.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
April 1, 1914.

A DISPLACEMENT OF ARC LINES NOT DUE TO PRESSURE.[2]

BY CHARLES E. ST. JOHN AND HAROLD D. BABCOCK.

WHEN the light from the different parts of the iron arc is examined by a powerful spectrograph, it is found that there are variations in the wave-lengths of the lines depending upon the portion of the arc used. A direct comparison has been made by us between the center of the iron arc and the region very near the negative pole. It has been extremely difficult and in fact practically impossible to obtain with certainty non-simultaneous photographs of spectra and comparisons that do not show minute relative displacements which must be referred to the instrumental conditions, even though the apparatus is very stable and the optical parts are in a constant temperature underground compartment.

By an arrangement of prisms over the slit of the plane grating spectrograph of 30-foot focus we now obtain *simultaneous* exposures, and are able to determine the absolute shifts between the pole and the center of the arc. There are some groups of lines—groups *a*, *b*, and *c* 4 of the Mount Wilson classification—that show no determinable displacements between the center and the negative pole, there are other groups, such as *c* 5, *d*, and *e*, the wave-lengths of which vary with the portion of the arc used, by such increments that, if pressure were the cause, a pressure difference of more than an atmosphere would be indicated between pole and center. The pressure shift for lines of group *b* in the red is 0.009 Å per atmosphere, as found by Gale and Adams. These lines are among the best lines in the iron spectrum, and they do not show displacements greater than the accidental errors of measurement which in the case of these lines do not amount to 0.001 Å.

It appears then that the displacements shown by the groups *c* 5, *d* and *e* between pole and center are not due to pressure. In general these lines are widened unsymmetrically at the negative pole, and the displacement was at first ascribed to the unsymmetrical broadening as the negative pole was approached, but the intensity curves obtained by Koch's *microphotometer* show that the maxima are displaced in the direction and by the amount

[1] Bulletin of the Bureau of Standards.

[2] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914.

indicated by the measured shift. Provisionally we are inclined to ascribe these displacements to increase of density, but the effects of other conditions are still under investigation.

The following table shows the results in the case of a few typical lines:

Negative Pole minus Center of the Arc.

Group *a*.		Group *b*.		Group *d*.		Group *e*.	
λ	*P*−*C*	λ	*P*−*C*	λ	*P*−*C*	λ	*P*−*C*
5,323	−0.0000Å.	6,136	+0.0007Å.	5,281	+0.018Å.	5,364	−0.022Å.
5,332	−0.0002	6,137	−0.0004	5,283	+0.019	5,367	−0.026
5,341	−0.0006	6,213	+0.0003	5,324	+0.022	5,369	−0.020
5,497	−0.0003	6,219	+0.0010	5,339	+0.016	5,383	−0.024
5,501	−0.0004	6,230	+0.0004	5,393	+0.017	5,400	−0.024
5,506	+0.0004	6,252	+0.0000	5,653	+0.014	5,410	−0.024
Mean.....	+0.0002Å.		+0.0003Å.		+0.018Å.		−0.023Å.
Displ. per atm....	+0.0036		+0.0094		+0.015		−0.018

MT. WILSON OBSERVATORY,
CALIFORNIA.

ON THE GROWTH AND DECAY OF COLOR SENSATIONS IN FLICKER PHOTOMETRY.[1]

BY M. LUCKIESH.

IT has been well established that photometric measurements obtained by the direct comparison and flicker methods do not agree in most cases. Under favorable conditions this disagreement is only a few per cent., but where there is a large color difference such as obtains when red or green lights are compared with "white" light or with each other the results by the two methods often differ by many per cent. A comparative study has been made of the two methods and a further study has been made of the phenomena involved in the flicker method in order to account for the difference between the results by the two methods. The results by both methods are of course primarily effected by the variations in the color vision of various observers. For instance observers having so-called normal color vision on comparing red or blue with "white" light obtain results differing by as much as 100 per cent. But the fact that the two methods give different results indicates that the different phenomena underlying these two methods are of importance.

When flickering lights are observed it is at once obvious that the rates of growth and decay of the color sensations are liable to be involved. Broca and Sulzer have shown that there is an overshooting of luminous sensation varying with the intensity and color of the light. The writer has used complementary colors, red and blue-green, in studying the phenomena involved in flicker photometry. These have been intercompared in various ways and also com-

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914.

pared with "white" light. It has been verified that the ratios of the intensities of these two lights differ by many per cent. by the direct comparison and flicker methods.

The maximum values of sensation produced by these colored lights differ considerably at all frequencies of flicker up to vanishing-flicker frequencies. The speed at which flicker disappears is always lower for the blue-green light for equal intensities as measured by the direct comparison method. When blue-green light is flickered upon a steady red field the speed at which flicker disappears is much lower than for the case of red light flickering upon a steady green field. In these cases the illuminations on the photometer screen are balanced for equality by the direct comparison method. The speed or frequency at which a balance is made with the flicker photometer is found to be lower than the speeds of vanishing flicker for the two lights. There is a considerably greater overshooting of luminous sensation in the case of the red light flickering upon a steady blue-green field than in the case of blue-green light flickering upon a steady red field.

It is also found that the vanishing flicker frequency depends upon the shape of the "illumination wave," that is upon the contour of flicker. This suggests a possibility that flicker photometers differing in principle may give different results. The results indicate that the importance of the different rates of growth and decay of color sensations have not been considered seriously enough in connection with flicker photometry. It seems that the differences between the results by the direct comparison and flicker methods can be largely accounted for in the above phenomena.

NELA RESEARCH LABORATORY,
NATIONAL LAMP WORKS OF GENERAL ELECTRIC COMPANY,
CLEVELAND, OHIO.

REVERSIBLE TRANSITIONS BETWEEN SOLIDS AT HIGH PRESSURES.[1]

BY P. W. BRIDGMAN.

A LARGE number of substances have been examined over the range from 0° to 200° and from atmospheric pressure to 12,000 kgm. per sq. cm. The reversible transitions from one solid to another which occur within this range have been studied, and the relation between pressure and temperature along the transition lines, the difference of volume between the different forms, and the latent heat of transition have been determined. Data have been obtained for the following substances: AgI, NH_4NO_3, HgI_2, C_2Cl_6, KNO_3, $KClO_3$, $AgNO_3$, KSCN, NH_4SCN, KNO_2, and $CsNO_3$. Of these NH_4NO_3, $KClO_3$ and KNO_2 have each one new form not known before, and KNO_3 has two new forms. Particularly striking are the very rapid curvature of the $AgNO_2$ line, and the fact that there is a maximum temperature of transition between the red and the yellow modifications of HgI_2. The results for these various substances show nearly all possible kinds of behavior. This is different

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914.

from the behavior on the transition curve from liquid to solid, where all substances give results of only one type.

THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY, CAMBRIDGE, MASS.

SURFACE LEAKAGE OVER INSULATORS.[1]

BY HARVEY L. CURTIS.

THE resistance between two conductors insulated by a solid dielectric usually depends very largely on the surface condition of the dielectric. If the insulator is in a humid atmosphere, condensation of moisture is known to take place on the surface and this increases the surface leakage. This paper will attempt to find a connection between the moisture condensed and the surface leakage.

The surface resistivity is defined as the resistance between two opposite edges of a surface film which is one centimeter square. The surface conductivity is the reciprocal of this. The surface resistance of most insulators varies greatly with the humidity of the surrounding air. In one case (hard rubber which had been exposed to the sunlight) the surface resistivity in very dry air was nearly one trillion (10^{12}) times as large as in very humid air. With waxy materials such as paraffin, ozokerite and beeswax, there is very little change with the humidity but all other insulators show marked changes. A factor of 10^5 or 10^6 between high and low humidities is not uncommon. It is to be expected that this change in surface resistivity is due to condensed moisture.

The thickness of the surface film of water upon quartz and glass has been determined by weighing. Knowing this thickness and the surface resistivity, the conductivity of the surface film is readily computed.

Ihmori[2] found the thickness of the water film on quartz by weighing. With cleaned specimens at about 90 per cent. humidity the thickness was about 0.5×10^{-6} centimeters. With uncleaned specimens the thickness was found to vary from 1.4×10^{-6} to 6.2×10^{-6}. The surface resistivity of quartz at 90 per cent. humidity which had been carefully cleaned was found to be about 10^{12} ohms per centimeter square. This gives the conductivity of the water of the film as 2×10^{-6} reciprocal ohms per centimeter cube, which corresponds to a high grade of distilled water. With the uncleaned specimens the surface resistivity was as low as 10^8 ohms per cm. square. Assuming that this corresponds to the thickest film found by Ihmori, the conductivity of the water is 0.0017 reciprocal ohms per centimeter cube. Since the quartz is insoluble, the increased conductivity is due to soluble salts which were present on the surface. If it were entirely due to sodium-chloride, there must have been 6×10^{-9} gram of the salt on each square centimeter.

In the case of glass, it was found that careful cleaning produced much less change in the surface resistivity than in the case of quartz. In fact with soft

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914.

[2] Wied Ann., 31, p. 1006; 1887.

German glass there was very little if any change. A number of samples were measured, the lowest value of 2×10^7 ohms being for a soft glass. Parks[1] has found the thickness of the water film on glass to be about 10^{-5} cm. Hence for soft glass, the conductivity of the water in the surface film is 5×10^{-3} reciprocal ohms per centimeter cube. This can be accounted for by the solubility of glass in water.

In the case of the waxy materials, water does not spread over the surface, but collects in drops. Hence we would not expect any change of surface resistivity with humidity. That such is the case has already been noted.

The presence of inorganic salts on the surface of insulators may not only lower the conductivity of the film, but it may also increase its thickness. This was shown by the measurements of Ihmori already quoted. Hence in the case of rubber which has deteriorated by exposure to the light, thereby forming considerable quantities of deliquescent salts on the surface, the resistivity changes through a very wide range due to changes of humidity.

An oil film has a much higher resistance than a water film, but in the case of the very best insulators such a film may lower the surface resistivity at low humidities. With moderate insulators, however, it may have a beneficial effect.

Bureau of Standards,
Washington, D. C.

A Significant Instance of Galvanometer Instability.[2]

By W. P. White.

A CERTAIN moving coil galvanometer, whose coil was very well balanced, showed for some time great steadiness while standing on a stone shelf. Subsequently, it showed in the same situation great sensitiveness to certain tremors of the building. There was no visible jiggling of the coil, but the slamming of doors, moving of heavy boxes, or pounding, in certain locations, caused a deflection often exceeding a millimeter at a meter's distance. The deflection was such as might have been produced by slight tipping of the galvanometer. It was a radial field instrument and, like most of its kind, very sensitive to changes of level. An investigation seemed to indicate that the shelf was too flexible, and the galvanometer was fastened to a stout iron bracket which was secured to two boards which had been built into the wall for such purposes. The trouble entirely disappeared, and for some time the galvanometer was almost perfectly steady again. Subsequently, the trouble reappeared as badly as before. An examination indicated that the wooden boards were not quite firmly fastened to the wall. The iron shelf was then fastened directly and firmly to the brickwork of the wall itself, and the trouble again disappeared. Conclusions: (1) It is clear that if the stone shelf had not been steady at first, or if the wooden boards had been loose from the wall at the

[1] Phil. Mag. (6), *5*, p. 517; 1903.

[2] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914.

outset, the real nature of the trouble would probably never have been discovered. The reasonable inference would have been drawn that the tremors of the building as such were too much for the galvanometer, and some unnecessarily elaborate protection would have been installed. (2) It is clear that moving coil galvanometers, at any rate those with a radial field, may suffer serious perturbations which are not the direct result of oscillations or jiggling of the coil. If in this case the perturbations had been accompanied by a small amount of jiggling, the latter would of course have been blamed. It is quite possible, therefore, that changes of level, weak fastening of shelves, and other similar preventable causes may now be responsible for much galvanometer disturbance, but are not recognized and remedied because some jiggling is also present and gets credit for the whole trouble. (3) This seems very likely to be the case where a Julius suspension of the old type, with three wires, is used, since the ordinary breezes of the room, acting on these fine wires, may produce slight irregular expansions and contractions, which cause changes of level, and therefore deflections. In such a case, the conclusion might easily be drawn that the deflections are due to the vibrations of the building, that they therefore show that the suspension is inadequate, and that no further help is possible, whereas the case may be quite otherwise. Of course, if any effect of this sort is suspected with a Julius suspension, a test can easily be made which will show whether it is present or not.

GEOPHYSICAL LABORATORY,
WASHINGTON, D. C.

HIGH TEMPERATURE MEASUREMENTS WITH THE STEFAN-BOLTZMANN LAW.[1]

BY C. E. MENDENHALL AND W. E. FORSYTHE.

IN view of the increasing use of extremely high temperatures (2000–3000° C.) the establishment of a reliable scale for this reason becomes increasingly important. Above 1550° C., the limit this far attained with the gas thermometer, the only method available is some form of radiation pyrometer. Of these we may use for a perfect radiator, either the one based on a measurement of the total radiation or one depending upon the spectral distribution. While in many cases the latter is more convenient to use, the former is the more sound from a theoretical standpoint, for the reason the Stefan-Boltzmann law has been chosen as defining the temperature scale.

This work may be considered in three parts: (1) A comparison of the Stefan-Boltzmann scale of temperature with the Day and Sosman scale between 1063° C. and 1549° C. (the melting points of gold and palladium). (2) A determination of the melting point of platinum in terms of the Stefan-Boltzmann scale. (3) A comparison of the optical temperature scale based on the application of Wien's Law to a pyrometer of the Holborn-Kurlbaum type with the Stefan-Boltzmann scale up to 2820° C.

Apparatus.—The optical pyrometers were calibrated by observations on a

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914.

black body furnace held at the melting points of palladium and gold respectively. For the intermediate points and for extending the scale rotating sectors of known aperture were used, combined with Wien's distribution law in which C_2 was taken as 14,500 and the wave-length used was 0.658 μ. To obtain the high temperature the carbon tube furnace described in the Astrophysical Journal, December, 1911, page 353, was used. The arrangement of this furnace was altered somewhat to allow for a series of water-cooled diaphragms that served both to protect the receiving thermocouple and also to limit the beam of radiation. The inside of the furnace was carefully washed out with nitrogen before any attempt was made to make any determination of temperature, by measuring the total radiation. The receiving thermocouple consisted of a single junction of Bi-Sb and Sb-Cd alloys, the hot junction being soldered to a light receiving disc of silver foil 3 mm. in diameter. This receiving disc was blackened with acetelyne smoke and mounted at the center of a hemispherical concave mirror. Under these circumstances the receiver has been assumed to be a perfect absorber, since only differential errors would be introduced if this assumption were not quite correct.

If a certain deflection D of the galvanometer connected to the thermopile, is obtained at a temperature T_1 (always the gold or palladium melting point) with clear aperture, and the same deflection is obtained through a rotating sector whose transmission is S (< 1) at a temperature T_2 then $T_2 = T_1/\sqrt[4]{S}$ which determines the temperature on the Stefan-Boltzmann scale.

Twelve comparisons were made of the Stefan-Boltzmann scale and the gas scale between 1063° C. and 1549° C. The total radiation sector having been cut in the ratio $(1{,}063 + 273/1{,}549 + 273)^4$ the ratio of the corresponding deflections should have been unity, the observed mean ratio was 1.001, the maximum ratio being 1.007, the minimum 0.998. Our value for the melting point of platinum depends upon three very concordant direct determinations of the melting point in a graphite tube black body furnace carried out by Mr. C. G. Peters. The platinum was enclosed in porcelain and protected from the action of the reducing gases by a flow of nitrogen. The average value was 1753° C. The temperature scale based on the Stefan-Boltzmann law and that based on the Wien Distribution law, were compared at the following points with the results indicated:

Number of Comparisons.	Temperature ° C.	$T_W - T_{S-R}$	Range of Observations.
9	1750	Less than ± 0.5° C.	
7	2200	Less than $+2$.	2° C
3	2500	About $+2$.	4
4	2820	About $+4$.	7

DEPARTMENT OF PHYSICS,
UNIVERSITY OF WISCONSIN.

COLD-END COMPENSATOR FOR THERMOCOUPLES.[1]

BY CHARLES B. THWING.

THE method of compensating for variations in the temperature of the cold ends of a thermocouple used in connection with a direct reading indicator or recorder consists in placing in series with the couple at the cold end a Wheatstone bridge of low resistance. Three arms of the bridge are of manganin, the fourth of nickel. The current through the bridge is so adjusted by resistance in the battery circuit as to give direct readings of temperature at the bridge when the thermocouple is not in circuit. When couple and bridge are connected in series the readings of bridge and thermocouple are combined to give true temperature readings. Otherwise expressed, the bridge automatically sets the zero of the indicator to cold end temperature. (The apparatus was exhibited, attached to a recording pyrometer.)

PHILADELPHIA, PA.,
March 24, 1914.

THE EMISSIVITY OF METALS AND OXIDES. I: NICKEL OXIDE (NiO) IN THE RANGE 600 TO 1300° C.[1]

BY G. K. BURGESS AND P. D. FOOTE.

THE monochromatic emissivity, E_λ, throughout the visible spectrum and the total emissivity, E, of nickel oxide (NiO) have been measured for the temperature interval 600 to 1300° C. The monochromatic emissivity increases linearly with increasing wave-length and decreases linearly with increasing temperature over the region investigated. For example, at 1160° C., E_λ increases from 0.865 at 0.5 μ to 0.882 at 0.7 μ, and for $\lambda = 0.65\,\mu$, E_λ decreases from 0.958 at 800° C. to 0.845 at 1300° C. The total emissivity increases with increasing temperature, but the relation is not linear. Temperatures and E have respectively values as follows: 600°, 0.54; 800°, 0.68; 1000°, 0.76; 1200°, 0.85; 1300°, 0.87.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

SOME EFFECTS OF DIFFRACTION ON BRIGHTNESS MEASUREMENTS MADE WITH THE HOLBORN-KURLBAUM OPTICAL PYROMETER.[1]

BY A. G. WORTHING AND W. E. FORSYTHE.

IT has generally been assumed that, when the pyrometer filament in a Holborn-Kurlbaum optical pyrometer apparently matched the background image in brightness, the filament and the image were at the same brightness. However, in attempting to compare temperature measurements on a tungsten filament made with an ordinary Holborn-Kurlbaum optical pyrometer with those made with a modified one as described below, such was found not to be the case.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914.

The apparatus was arranged as indicated in Fig. 1 where we have shown also the dimensions. It was found that varying the size of the opening at C, that is varying the angle α, keeping, however, the angle α greater than the angle β, necessitated a change in the brightness of the pyrometer filament in order to maintain the apparent match in brightness with the background image. In a special case, keeping all other parts of the apparatus constant, and varying only the angle α, an apparent change in the brightness of the background image of 60 per cent. was caused. Other conditions being the same, the smaller the angle and the larger the pyrometer filament, the brighter the background image appeared as compared with the brightness of the pyrometer filament. When the angle α was changed by varying the opening in C only in a direction perpendicular to the axis of the pyrometer filament, the apparent change in brightness was the same as when the aperture in C was increased in all directions.

On blocking out the central portion of the objective lens so that, from geometrical considerations only, no light from the background in the immediate neighborhood of the point where the pyrometer filament is seen projected, can enter the eyepiece opening, the filament is still seen apparently illuminated, even though no current is passing through it. When an eye piece of high enough resolving power was used, this brightness of the pyrometer filament was seen to consist of bright streaks along its edge. Because of these and other effects it was decided that the apparent change in brightness of the pyrometer filament was due to diffraction of the light from the background at the pyrometer filament.

Some necessary working conditions for avoiding difficulties arising from diffraction and other causes in optical pyrometry are added:

1. The sources to be studied should be used as backgrounds and not as pyrometer filaments.

2. A single pyrometer filament should be used throughout an intercomparison.

3. The angles α and β should be definitely fixed. This can be best done by having limiting diaphragms at C and E which are at fixed distances from the pyrometer filament, preferably as far away as possible.

4. The apparatus should be so adjusted that there is approximate axial symmetry.

5. The resolving power of the eyepiece should not be so great as to prevent the practical disappearance of the pyrometer filament against the background image.

6. The image of the background should be large in comparison with the pyrometer filament.

7. The magnifying power of the eyepiece should be sufficiently large so that no difficulty is experienced by the observer in fixing on the intersection and so that the effects due to the eye's imperfections are largely eliminated.

If in addition, the following desirable conditions are fulfilled, it will be found that brightness matches can be more easily obtained.

1. The eyepiece should have as high a resolving power as possible, but still with the condition that when the pyrometer lamp filament is balanced in brightness against the background, there will be disappearance.

2. An eyepiece and eyepiece diaphragm combination which gives comparatively large clear images is of course desirable.

3. Considerable depends upon the pyrometer lamp. In general the writers have found filaments in either hairpin loops or single straight filaments mounted in spherical bulbs to be the most satisfactory. In general the larger bulbs have been found more free from imperfections than the smaller bulbs.

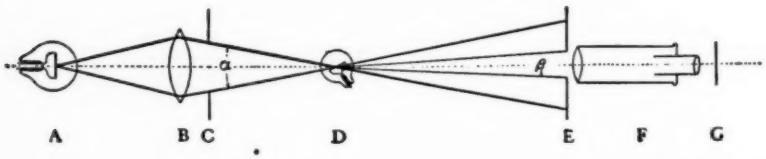

Diagram showing arrangement of apparatus. *A*, background; *B*, objective lens; *C*, entrance cone diaphragm; *D*, pyrometer filament; *E*, eyepiece diaphragm; *F*, eyepiece; *G*, monochromatic glass filler. $AB = 25$ cm., $BD = 128$ cm., $DE = 185$ cm. Diameter of aperture at *E*, 9 mm.

4. For the sake of ease of adjustments, particularly in obtaining axial symmetry, it is very desirable to have the diaphragm *C* one with a variable aperture.

5. In cases where high accuracy is demanded a gain may sometimes be made by using light of the shorter wave-lengths.

NELA RESEARCH LABORATORY,
NATIONAL LAMP WORKS OF GENERAL ELECTRIC COMPANY,
CLEVELAND, OHIO.

THE RESULTS OF THE ATMOSPHERIC ELECTRIC OBSERVATIONS ON THE SECOND CRUISE OF THE "CARNEGIE", JUNE, 1910, TO DECEMBER, 1913.[1]

BY C. W. HEWLETT.

A SYSTEMATIC series of observations extending over three years has been made on the potential gradient, conductivity, and the radioactivity of the atmosphere during fifteen passages in the Atlantic, Indian, and Pacific Oceans. The results will be elsewhere analyzed and treated in detail. The mean values for the conductivity vary considerably in the different Oceans, but in all cases were high in comparison with the results previously found on land and on sea. The values were of the order of from 50 to 100 per cent. greater than those usually found for land and seemed to show very little connection with the radioactivity of the atmosphere. Only in the Atlantic Ocean was an appreciable amount of radioactivity observed. The average activity here being 8 (in Elster and Geitel units).

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914.

The average value for the potential gradient in the Atlantic Ocean was about equal to that in the Pacific and about 60 per cent. greater than the value in the Indian Ocean. With the exception of one observation the potential gradient was always positive when even the observations were made during rain. The relations of the elements among themselves and with the various meteorological factors is in accordance with what would be expected from theoretical considerations.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington.

A Sensitive Moving-Coil Galvanometer.[1]

By F. Wenner, E. Weibel and F. C. Weaver.

THE galvanometer is designed for use in bridge, potentiometer, and similar work where the resistance of the apparatus is fairly low and a high sensitivity is required. To distinguish the design described here from others we shall refer to it as type *m*.

The coil is 8 × 12 mm. in mean area and has 27½ turns of .080 mm. (No. 40 American wire gauge) single silk-covered copper wire. It is wound in such a way that there is a loop at both the top and bottom to which the suspensions and mirror are attached. A second loop is brought out both at the top and the bottom and extends to the front. These with a wire added later form a truss to stiffen the coil. Before the coil is removed from the form on which is is wound it is treated chemically to remove all traces of iron in the insulation and on the surface of the wire. After thorough washing and drying the coil, still on the form, is dipped in a thin solution of collodion which when dry binds the winding firmly together.

The mirror is 1 cm. in diameter and .6 mm. in thickness.

The suspensions are made from .015 mm. copper wire rolled flat. They are each about 4 cm. long and by means of a spring are held taut, the tension being of the order of 1,000 dynes.

The point to which the lower end of the lower suspension is attached is not directly under the point to which the upper end of the upper suspension is attached, but is from 1 to 3 mm. to the front. Thus the coil rotates about an axis making a small angle with the vertical. In balancing the coil the adjustment is made so that the center of mass is a fraction of a mm. to the front of the axis of rotation. This gives the moving system a small gravity control opposing the control of the suspensions. By changing the level of the instrument the gravity control may be changed and consequently the period of the moving system changed as desired.

The iron core is 6 mm. in diameter and 1 cm. long. The pole pieces are of the same length as the core and have a radius of 4 mm. In assembling the spacing is such that the air gaps are 2 mm. at the center and slightly less under the pole tips, making the field very nearly radial.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914.

A strong magnet is used and it is provided with an adjustable shunt so that the intensity of the field in the air gap can easily be varied over a considerable range.

With the period adjusted to 10 seconds and the field strength adjusted so as to make the external critical resistance 20 ohms the sensitivity is approximately 20 mm. per micro-volt, with the scale at a distance of 1 meter.

As compared with the most sensitive "low resistance" galvanometers on the market, taking the mean of the operating constants of three instruments of different designs, we have, approximately—external critical resistance 65 ohms —period 6 seconds—and sensitivity 2 mm. per micro-volt.

It is probable that further experience with this design will suggest minor changes in the construction.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

THE CONTROL OF THE WAVE-LENGTH-SENSIBILITY CURVES FOR SELENIUM.[1]

BY E. O. DIETERICH.

AN investigation of the conditions governing the shape of the sensibility curves for selenium led to the production of several new types of cells, which may be classified into two general groups; those that have a maximum sensibility for wave-lengths longer than 640 $\mu\mu$, and those that show very little sensitiveness in this region, but have a maximum for wave-lengths shorter than 640 $\mu\mu$. The location of the maximum for the different types produced thus far is as follows: 440 $\mu\mu$, 500 $\mu\mu$, 550 $\mu\mu$, 700 $\mu\mu$, 720 $\mu\mu$, and 800 $\mu\mu$. Those that show a maximum in the red also have a pronounced minimum at 640 $\mu\mu$, and a broad maximum at the shorter wave-lengths. Some, however, have two well-defined maxima; those that have a maximum at 440 $\mu\mu$ also show one at 700 $\mu\mu$ or 720 $\mu\mu$, and a minimum at 640 $\mu\mu$.

The location of the maximum seems to be determined entirely by the method of "annealing" the cells. The samples annealed at 210° C., or higher, all show a maximum below 640 $\mu\mu$, and no maximum at all in the longer wave-lengths. Those annealed at lower temperatures, 180° C., all show a maximum at wave-lengths greater than 640 $\mu\mu$, which is higher than the broad, undefined maximum in the shorter wave-lengths. When the samples are annealed at temperatures between 180° C. and 210° C., the height of the maximum in the red, relative to that in the blue, becomes less and less, until at about 210° C. it disappears. Further work, to determine more accurately the governing conditions, is in progress.

THE PHYSICAL LABORATORY,
STATE UNIVERSITY OF IOWA.

THE TESTING OF POTENTIOMETERS.[1]

BY FRANK WENNER AND ERNEST WEIBEL.

FOR precise measurements with a potentiometer the errors in the adjustment of the resistance sections or coils in the apparatus must be known.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914. To be published in the Bulletin of the Bureau of Standards and reprinted as Scientific Paper No. 223.

These errors can be determined by measurements of the resistances of the various sections but, without some systematic method of applying corrections, are not easily allowed for, in the use of the potentiometer.

We have found that it is convenient to express the relation between the known electromotive force, the unknown electromotive force, the readings and the corrections by the formula

$$E = f[(e_1 + a_1) + (e_2 + a_2) + \text{etc.}]\ (1 + b + c + d) \qquad (1)$$

where E is the value of the unknown electromotive force

f is the range reading,

e_1 is the reading of the highest dial,

e_2 is the reading of the next highest dial, etc.,

$$c = \frac{S - s}{s},$$

where S is the value of the known electromotive force and s is the reading of the known electromotive force dial or dials, and b, d, a_1 and a_2, etc., are small corrections due to errors in the adjustment of the various resistance sections. The correction a_1 depends only on the reading e_1, a_2 depends only on e_2, etc.,

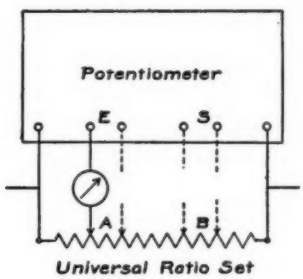

Fig. 1.

b depends only upon the reading s and d depends only upon f. A table can therefore be constructed for each dial giving the corrections corresponding to its various readings.

Let R_e be the resistance in the potentiometer between the E terminals and R_s that between the S terminals. Then if the total current is independent of the settings of the various switches, plugs, etc.,

$$E = SR_e/R_s = f[(e_1 + a_1) + (e_2 + a_2) + \text{etc.}]\ (1 + b + c + d). \qquad (2)$$

Thus if R_e/R_s is determined for the various readings of f, of e, and of s the corrections b, d, a_1, a_2, etc., can be determined.

In order to measure R_e/R_s readily to a reasonably high accuracy, a special piece of apparatus was designed and has been in use during the past two years. This apparatus, which we have designated a "Universal Ratio Set," is equivalent to 211110 resistance coils, of .01 ohm each, connected in series (connection to the terminals of any coil being possible). This is accomplished with 100 coils

and 5 dial switches of the usual construction. If this apparatus be connected in parallel with the potentiometer to be tested a Matthiessen Hockin bridge is formed.

Now if for any setting of the potentiometer four balances are made as indicated in the figure we have

$$Re/R_s = A/B \qquad (3)$$

The value of A/B is readily determined from the readings and known corrections to the ratio set. If such measurements are made for all settings of each of the dial switches, range switches or plugs, etc., we have data from which all the corrections of equation (1) can be calculated. In use certain adjustments are made (which cannot be explained in a short abstract) such that the calculations are very simple. The apparatus has been found to furnish a rapid and direct means for calibrating any type of potentiometer.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

SOME RECORDS OF THE WIRELESS TIME SIGNALS MADE WITH A PHYSIOLOGICAL RECORDER.[1]

BY C. W. WAGGONER.

THE intensity of the wireless time signals from Arlington, Va., which are received on a small aerial at W. Va. University, led the writer to try a muscle-nerve preparation of a frog and it was found that this preparation responds very well to the stimulus sent from the navy station.

The sciatic nerve was connected to the receiving circuit in place of the high resistance telephones, the gastrocnemius muscle having been attached to a long lever operating over a smoked-drum kymograph. A number of complete five-minute records showing the fatigue of the muscle. Records of the weather reports were made, but the speed was too high to make it possible to follow with this type of recorder.

MORGANTOWN, W. VA.,
March 23, 1914.

AN ELECTROMAGNETIC PUZZLE.[1]

BY F. J. ROGERS.

IF an electric current flows through a long straight tube the return part of the circuit being at a great distance, or still better, symmetrically distributed on opposite sides of the tube, there will be *no magnetic field* inside of the tube at any time whether the current is constant or varying at any rate.

If a wire threads through the tube there will be an induced current in it when the current in the tube varies. This induced E.M.F. is proportional to the length of the tube and is equal (though slightly greater) to the induced E.M.F. in a wire just outside of the tube and very close to it.

This is a case of induction which is evidently not the same as is in mind when a conductor sweeps across lines of magnetic flux or when lines of magnetic flux are thought of as sweeping across a fixed conductor.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914.

In our case there is no magnetic field at any time inside the tube where the induced E.M.F. exists.

How are we to look upon this case of induction? To those who are completely satisfied with the statement that the induced E.M.F. in a circuit is equal to the rate of variation of the flux through the surface of which this circuit is the contour nothing more need be said. For this case of induction is perfectly in accord with that statement. To those who admit that the above statement is only short-hand mathematics and not physics it may be worth while to say that this statement ascribes an effect (the induced E.M.F. in a wire inside a tube) to a cause (a variation in magnetic flux outside of the tube) which does not exist where the effect is produced.

Can we save our face in this matter and still preserve the imagery of an induced E.M.F. being produced by lines of magnetic flux sweeping across the fixed conductor? I think we may do so in the following manner: Suppose when the current in the tube is being built up from a zero value that closed lines of flux of molecular dimensions originate in the *substance* of the tube and expand with rising current until they surround the tube. In expanding each line must cut across the tubular space and in doing so contribute its part to the induced E.M.F. in the interior of the tube. This method of imagining the process of induction will probably appear satisfactory to many but still the difficulty remains that induction takes place inside the tube while the resultant field is zero everywhere and at all times. The flux lines which we have imagined to be snapping across the tube come symmetrically from all sides in such a way that they mutually cancel each other with regard to the production of a magnetic field. Here enters an idea which seems to me worth consideration. Inside the tube we have a vector field which may be looked upon as the sum of two equal and opposite vectors and therefore a field of zero intensity. However, if these two equal and opposite components are varying, their inductive effects are arithmetically additive and a zero magnetic field is not a field of nothingness but still possesses electromagnetic properties.

I am perfectly aware of the fact that the induction in the case of a transformer in which the primary is uniformly wound on a ring and is overwound uniformly by the secondary is of exactly the same character as the case of induction I have been discussing. The case of the tube was the form in which this "puzzle" first appealed to me, but the fact that this condition occurs in such a familiar case does not make it any the less a puzzle.

PRINCETON UNIVERSITY.

NOTE ON THE PHYSIOLOGICAL EFFECT OF THE CURRENT.[1]

BY F. J. ROGERS.

FOR some years I have regularly required students to measure the resistance of their own bodies when current flows from one hand to the other, and to observe the maximum current, from a direct current source, that they were

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914.

willing to endure. Of course the resistance depends greatly on the character and area of contact. The maximum current which the average person is willing to endure proves to be not greatly different in different individuals. If the whole hand is immersed in water or if the electrodes are large, metal handles or plates grasped firmly, then practically every one is willing to endure 20 milliamperes of direct current, on the other hand I have seen not more than one or two who would not say enough before the current reached 40 milliamperes.

Even this difference is principally due, on the one hand to those who are disinclined to endure a little physical pain and on the other to those, who through emulation or otherwise, will not cry enough until their pain has become rather intense.

I have had this experiment performed on scores of students before paying heed to the very occasional remark that the pain was more intense in one arm than in the other. Finally with fewer students to pass through the ordeal and coming across one who was more insistent on the difference in sensation in the two arms or wrists, I reversed the current, when he declared the pain was now greater in the other wrist. With this beginning I proceeded to extend operations upon every one I could drag up to be tested, with uniformly the same result for all persons, male and female, old and young. This difference is not an absolute one. When the sensation is mild there is practically no difference in sensation according to the direction of the current, but just as soon as the painful sensation becomes rather intense the victim uniformly says that the pain is decidedly greater in one wrist or arm than in the other and this always proves to be the cathode arm, the one out of which the current is flowing.

The same condition prevails when the current flows from one finger to the corresponding finger of the other hand. In this case the pain is most intense in the cathode finger although still more or less distant from the electrode. If, however, the electrodes are in contact with the little finger of one hand and the thumb of the other hand, the pain is more intense in the little finger, no matter in which direction the current flows.

Without having attempted experimentation on others in this regard I at least know in my own case that when the current flows from one foot to the other the pain is more intense in the cathode ankle than in the other one.

When current flows through a moderately small electrode in contact with a sensitive portion of the skin, as for example the arm, the pain, in this case a sort of burning sensation, is always more intense at the surface immediately in contact with the electrode; this is well known but is not the fact which I have been trying to describe. It is not however well known to physiologists that when current flows through electrodes in contact with the outside of the face that a decided taste sensation is caused in the proper place for such sensations. I tried the same experiment with sugar and salt in my mouth (on separate occasions) to see whether epicures might possibly derive greater pleasure from such stimuli, however with negative results.

It is well known among physiologists that when a nerve is stimulated by an electric current, the sensation is more intense at the cathode than at the anode. However in my experiments with a large electrode in contact with the palms of the hands practically no sensation was produced at either electrode while at a distance from the electrode, namely in the wrist and forearm the sensation, when it become intense, depended upon the direction of the current, being more painful in the cathode wrist.

This fact which the physiologists seem not to know about may be harmonized with the greater sensitiveness at the cathode by making use in our explanation of what are called physiological cathodes. Wherever current flows from nerve tissue across a surface of separation into another kind of tissue such as muscle or tendon we have a change of material in the path of the current and physiologists call this a physiological cathode. Now I am told that in the wrist there are a great many sensory nerve endings which terminate in tendons. If these are called physiological cathodes when the corresponding hand is the cathode, the greater pain in the cathode wrist is brought into line with the greater sensitiveness of the cathode in nerve stimulation.

PRINCETON UNIVERSITY.

PHOTOGRAPHS OF RETROGRADE RAYS, (*a*) FROM THE COLD CATHODE, (*b*) FROM THE HOT LIME CATHODE.[1]

BY O. H. SMITH.

AT the Thanksgiving, 1912, meeting of the American Physical Society several photographs were exhibited by Knipp showing retrograde rays. Thomson had photographed them, however with limited detail, a few months before. The present paper deals with a more extended photographic study of these rays using both the cold and the hot lime cathodes. The apparatus employed is essentially that described by Knipp.[2]

(*a*) The photographs, in the case of the cold cathode, with very weak deflecting fields, show in a very striking manner the three kinds of carriers—the electron, the carriers atomic in size with a negative charge, and carriers atomic in size with a positive charge. For a given magnetic field these heavy ions show a deflection that is 2 or 3 per cent. of that of the electron. For strong deflecting fields the electrons are blown off the plate and in place of the single line representing each the heavy positively and negatively charged ions there appear a number of parabolic lines characteristic of the gas in the discharge vessel. Under favorable conditions some of the negative lines are duplicated on the positive side, *i. e.*, a negative line having a corresponding positive component.

(*b*) For the case of the hot lime the problem presents additional difficulty. First, in lining up the cathode, and second, apparently the heavy carriers do not possess sufficient energy to affect the photographic plate upon which they

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914.

[2] Phil. Mag., December, 1911; PHYS. REV., XXXIV., March, 1912.

fall. The lining up of the cathode is accomplished by mounting it upon a sort of universal joint which is actuated from without by means of taps. Even then the direction of the cathodic beam changes with the heating current and destroys the alignment necessitating continual adjustment. The energy of the ions is increased by passing them through an accelerating field. With very weak deflecting fields and with an accelerating gradient of about 2,500 volts per centimeter the photographic plates show both the electrons and the negative ions. When the accelerating field is reversed the positive ions are revealed and, of course, the electrons and negative ions in turn are suppressed. The retrograde rays show but very indistinctly even for the largest accelerating fields employed which was about 3,000 volts per centimeter. Their distinctness on the photographic plate depends in a marked way upon the electrical conditions within the discharge vessel, for instance the positive ions show best on the plate when the metallic parts within are grounded while the negative ions are more distinct when the connection within is to the anode. The photographs indicate that the positive rays are more intense than the negative ones.

A number of photographs showing the deflections under the above named conditions accompany the paper.

PHYSICAL LABORATORY, UNIVERSITY OF ILLINOIS.

AN EXTENSION OF THE SPECTRUM IN THE EXTREME-VIOLET.[1]

BY THEODORE LYMAN.

THE researches of Schumann led him to extend the spectrum to the neighborhood of wave-length 1,250. His limiting wave-length was determined by the absorption of the fluorite which formed a necessary part of his apparatus. In 1904, I succeeded in pushing the limit to wave-length 1,030 by the use of a concave diffraction grating.

Recently I have renewed the attack on the problem with the result that I have succeeded in photographing the spectrum of hydrogen to wave-length 905. The extension is due not so much to any fundamental change in the nature of the apparatus as to an improvement in technique consequent on an experience of ten years.

It is a characteristic of the region investigated by Schumann between wave-lengths 1,850 and 1,250, that while hydrogen yields a rich secondary spectrum, with the possible exception of one line, no radiation has been discovered belonging to the primary spectrum. On the other hand, in the new region between the limit set by fluorite and wave-length 905, a disruptive discharge in hydrogen produces a primary spectrum of great interest made up of perhaps a dozen lines. These lines are always accompanied in pure hydrogen by members of the secondary spectrum but they may be obtained alone if helium containing a trace of hydrogen is employed.

Results obtained from vacuum tubes when a strong disruptive discharge is used, must always be interpreted with caution since the material torn from the

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914.

tube itself sometimes furnishes impurities. In the present case, it will be some time before the effect of such impurities can be estimated. However, it may be stated with some degree of certainty that the diffuse series predicted in this region by Ritz has been discovered. The first member at 1,216 is found to be greatly intensified by the disruptive discharge and the next line at 1,026 appears also, though very faintly. This diffuse series bears a simple relation to Balmer's formula. Following the same kind of argument, a sharp series corresponding to the Pickering series might be expected. The new region appears to yield two lines belonging to such a relation at positions demanded by calculation.

Jefferson Physical Laboratory, Harvard University,
April 20, 1914.

ERRATUM.

Pages 50 and 52. In the paper by A. K. Ångström entitled "Some Observations on Selective Reflection from Solutions in the Infra-red," cuts 1 and 3 should be interchanged; as printed these cuts and the descriptive captions underneath do not corrrespond.

Index to Volume III., Series II.

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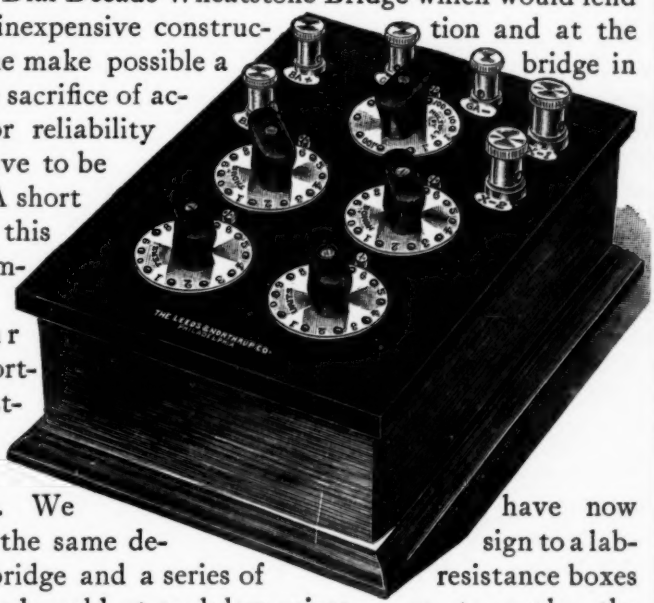

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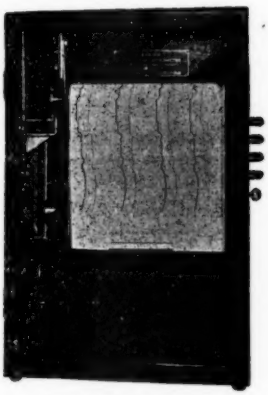

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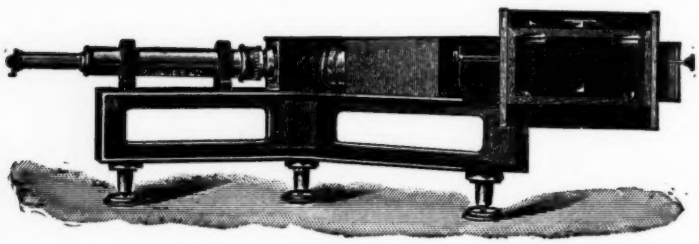

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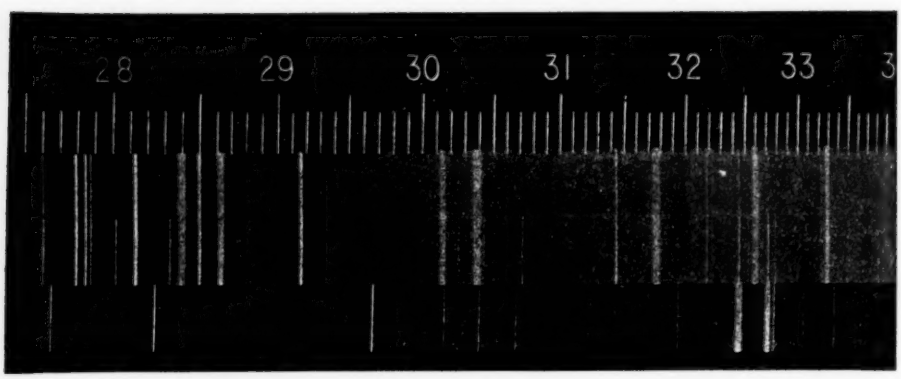

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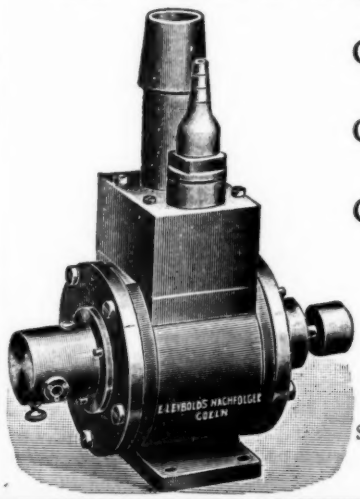

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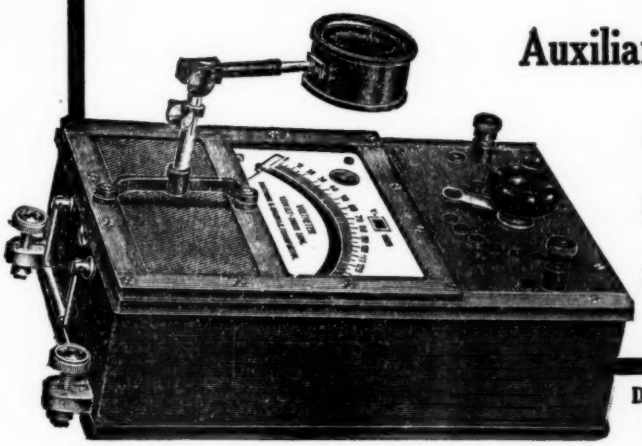

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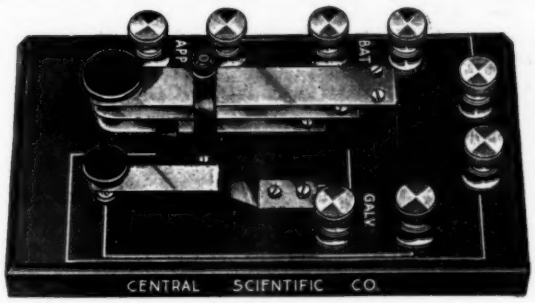

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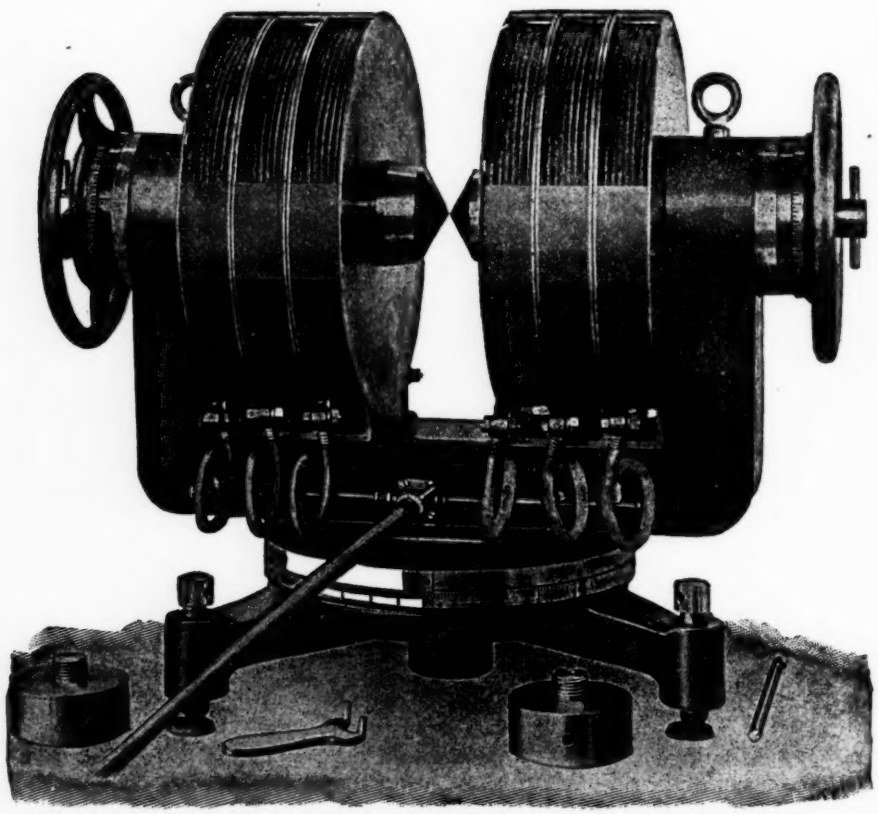

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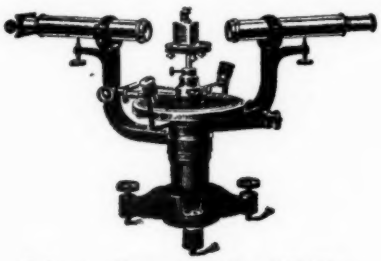

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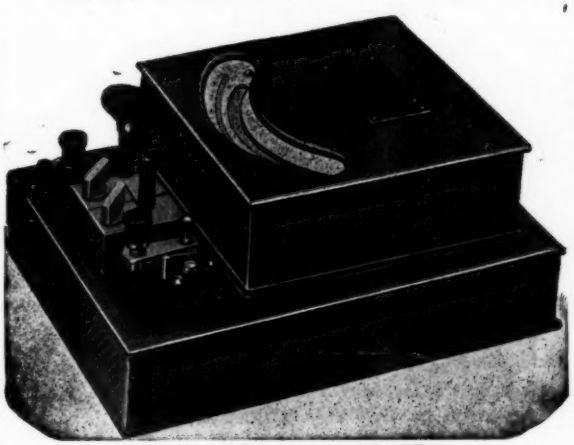

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